STATUS AND FUTURE OF $\gamma$

J. Rosner - Beauty 2003 - CMU 10/14/2003

Thanks to C.-W. Chiang, M. Gronau, Z. Luo, D. Suprun for enjoyable collaborations

Separating strong and weak phases in 2-body $B$ decays

$\beta$ from $B \to J/\psi K_S$ vs. $\alpha$ from $B^0 \to \pi^+\pi^-$

CP-averaged decay rates and CP asymmetries: $B \to K\pi$ examples

Analysis of $B \to VP$ decays within flavor SU(3)

$B \to PP$ decays: Results involving $\eta$ and $\eta'$

$B \to DK$ decays using $D \to CP$ eigenstates

The Unitarity Triangle

\[
\begin{array}{c}
\phi_3 = \gamma \\
\phi_2 = \alpha \\
1 - \rho - i\eta \\
\rho + i\eta \\
(\rho, \eta)
\end{array}
\]
$B^0 \to J/\psi K_S$ VERSUS $B^0 \to \pi^+ \pi^-$

$B^0 \to J/\psi K_S$: one main subprocess $\bar{b} \to \bar{c}c\bar{s}$

Direct decay interferes with $B^0 \to \bar{B}^0$ mixing $(e^{-2i\beta})$

Provides $\sin 2\beta = 0.736 \pm 0.049$ without much ambiguity

$B^0 \to \pi^+ \pi^-$: Two types of amplitude, “T” (tree) and “P” (penguin)

Different weak and strong phases can complicate the analysis

Tree only: direct $A(B^0 \to \pi^+ \pi^-) \sim e^{i\gamma}$ interfering with $A(B^0 \to \bar{B}^0) \sim e^{-2i\beta}$

$\times A(\bar{B}^0 \to \pi^+ \pi^-) \sim e^{-i\gamma}$ to measure relative phase $2(\beta + \gamma) = 2\pi - 2\alpha$

So in absence of penguin, measure $\alpha$

Seek estimate of $|P/T|$ or observables not requiring this ratio

Mass eigenstates: $B^0_L = p|B^0\rangle + q|\bar{B}^0\rangle$ ; $B^0_H = p|B^0\rangle - q|\bar{B}^0\rangle$

$q/p = e^{-2i\beta}$ , $\lambda \equiv (q/p)(\bar{A}/A)$ , $A \equiv A(B \to f)$ , $\bar{A} \equiv A(\bar{B} \to \bar{f})$

Observables in time-dependence of \begin{bmatrix} B^0 & \bar{B}^0 \end{bmatrix} \to \pi^+ \pi^- (or other final state):

$\Gamma(t) \sim e^{-\Gamma|t|}[1 \mp S \sin \Delta mt \mp A \cos \Delta mt]$, $t \equiv t_{\text{decay}} - t_{\text{tag}}$

$S = \frac{2\text{Im}\lambda}{1+|\lambda|^2}$ , $A = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} = A_{CP} = \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)}$
ASYMMETRIES IN $B \rightarrow \pi^+\pi^-$

<table>
<thead>
<tr>
<th>Observable</th>
<th>BaBar</th>
<th>Belle</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\pi\pi}$</td>
<td>$-0.40 \pm 0.22 \pm 0.33$</td>
<td>$-1.23 \pm 0.41^{+0.08}_{-0.07}$</td>
<td>$-0.58 \pm 0.20$</td>
</tr>
<tr>
<td>$A_{\pi\pi}$</td>
<td>$0.19 \pm 0.19 \pm 0.05$</td>
<td>$0.77 \pm 0.27 \pm 0.08$</td>
<td>$0.38 \pm 0.16$</td>
</tr>
</tbody>
</table>

With no penguins: $S_{\pi\pi} = \sin 2\alpha < 0$ would favor $\alpha > 90^\circ$

With penguins estimated from $B \rightarrow K\pi$ ($|P/T| \simeq 0.3$, M. Gronau + JLR):

Also useful: $R_{\pi\pi} = \frac{\Gamma(B^0 \rightarrow \pi^+\pi^-)}{\Gamma(B^0 \rightarrow \pi^+\pi^-)_{\text{tree}}} = 0.87^{+0.11}_{-0.28}$ (Z. Luo + JLR)

With $S_{\pi\pi}, A_{\pi\pi}, R_{\pi\pi}$ solve for weak & strong phases, resolve discrete ambiguity.
\[ B \rightarrow K \pi \] INFORMATION

Use decay rates averaged over CP and direct CP asymmetries

Probe tree-penguin interference in various processes (prime: \(|\Delta S| = 1\)):

\[ B^0 \rightarrow K^+\pi^- (T' + P') \] vs. \[ B^+ \rightarrow K^0\pi^+ (P') \] (Fleischer-Mannel, Gronau-JLR)

\[ B^+ \rightarrow K^+\pi^0 (T' + P' + C') \] vs. \[ B^+ \rightarrow K^0\pi^+ (P') \] (Neubert-JLR)

\[ B^0 \rightarrow K^0\pi^0 \] vs. other modes (Buras-Fleischer; Beneke-Neubert)

Using flavor SU(3) (often only U-spin, i.e. \( s \leftrightarrow d \))

Example: \[ B^0 \rightarrow K^+\pi^- \]

Tree \( T' \sim V_{us}V_{ub}^* \), phase \( \gamma \); penguin \( P' \sim V_{ts}V_{tb}^* \), phase \( \pi \); define \( r \equiv |T'/P'| \)

\[
A(B^0 \rightarrow K^+\pi^-) = |P'|[-1 + re^{i(\gamma+\delta)}], \quad A(\bar{B}^0 \rightarrow K^-\pi^+) = |P'|[-1 + re^{i(-\gamma+\delta)}],
\]

\[
A(B^+ \rightarrow K^0\pi^+) = A(B^- \rightarrow \bar{K}^0\pi^-) = -|P'| \text{ (no tree)}
\]

Test: no \( A_{CP} \) in \( B^+ \rightarrow K^0\pi^+ \) (or in \( B^+ \rightarrow \bar{K}^0K^+ \), where it would be bigger)

\[
R \equiv \frac{\Gamma(B^0 \rightarrow K^+\pi^-) + \Gamma(\bar{B}^0 \rightarrow K^-\pi^+)}{2\Gamma(B^+ \rightarrow K^0\pi^+)} = 1 - 2r \cos \gamma \cos \delta + r^2
\]

Fleischer-Mannel: \( R \geq \sin^2 \gamma \) for any \( r, \delta \) so if \( 1 > R \) then get a useful bound

Better: use also \( RA_{CP} = -2r \sin \gamma \sin \delta \); eliminate \( \delta \)
\[ B^0 \rightarrow K^+ \pi^- \] \text{ CONSTRAINTS}

\[ R < 1 \text{ at the } 1\sigma \text{ level: Obtain an upper bound } \gamma < 78^\circ \]

Inputs: \( R = 0.898 \pm 0.071, \ A_{CP} = -0.095 \pm 0.029, \ r = |T'/P'| = 0.12^{+0.04}_{-0.02} \)

Most conservative bound arises for smallest \( A_{CP} \), smallest \( r \):

Allowed region lies between the curves \( A_{CP} = 0 \) and \( |A_{CP}| = 0.124 \ (1\sigma) \)
SOURCE OF INPUTS

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Amplitude</th>
<th>$B$ (units of $10^{-6}$)</th>
<th>$A_{CP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow K^0\pi^+$</td>
<td>$P'$</td>
<td>$21.8 \pm 1.4$</td>
<td>$0.016 \pm 0.057$</td>
</tr>
<tr>
<td>$B^+ \rightarrow K^+\pi^0$</td>
<td>$-(P' + C' + T')/\sqrt{2}$</td>
<td>$12.8 \pm 1.1$</td>
<td>$0.00 \pm 0.12$</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^+\pi^-$</td>
<td>$-(T' + P')$</td>
<td>$18.2 \pm 0.8$</td>
<td>$-0.095 \pm 0.029$</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^0\pi^0$</td>
<td>$(P' - C')/\sqrt{2}$</td>
<td>$11.9 \pm 1.4$</td>
<td>$0.03 \pm 0.37$</td>
</tr>
</tbody>
</table>

Cheng-Wei Chiang et al., in preparation, based on Heavy Flavor Averaging Group

How to estimate the tree amplitude and $r = |T'/P'|$:

The tree dominates $B^0 \rightarrow \pi^+\pi^-$ (also $P$) and $B^+ \rightarrow \pi^+\pi^0$ (also $C$)

Factorization gives $T$ from $B^0 \rightarrow \pi^-\ell^+\nu_\ell$ at low $q^2$

Then one uses $\left| \frac{T'}{T} \right| = \frac{f_K}{f_\pi} \left| \frac{V_{us}}{V_{ud}} \right| \simeq (1.22)(0.23) = 0.28$

Validity of SU(3) flavor symmetry

SU(3) breaking taken into account in ratio of tree amplitudes

No breaking taken in other amplitudes: Don’t trust factorization for $C$ or $P$

Assumed same relative tree-penguin strong phases for $|\Delta S| = 1$ and $\Delta S = 0$.

Tests from penguin-dominated $B \rightarrow K\bar{K}$ decays, CP asymm. relns., $B_s$ decays
$B^+ \rightarrow K^+\pi^0$ CONSTRAINTS

Ratio $R_c \equiv \frac{2\Gamma(B^+ \rightarrow K^+\pi^0)}{\Gamma(B^+ \rightarrow K^0\pi^+)}$ also can deviate from 1

Non-penguin contributions include $T' + C'$ and electroweak penguin $P'_{EW}$

Details by M. Neubert and JLR; Neubert JHEP; JLR hep-ph/0306284

$R_c$ at the $1\sigma$ level: Obtain a lower bound $\gamma > 44^\circ$

Inputs: $R_c = 1.18 \pm 0.12$, $A_{CP} = 0.00 \pm 0.12$, $r_c = |(T' + C')/P'| = 0.195 \pm 0.016$

Most conservative bound arises for smallest $A_{CP}$, largest $r_c$, largest $|P'_{EW}|$:
$B^0 \rightarrow K^0\pi^0$ CONSTRAINTS

\[ R_n \equiv \frac{\Gamma(B^0\rightarrow K^+\pi^-)}{2\Gamma(B^0\rightarrow K^0\pi^0)} = \left| \frac{p'+t'}{p'-c'} \right|^2 = 0.76 \pm 0.10 \]

Should be same to leading order (small letters: include EWP; bar: CP avg.) as

\[ R_c \equiv \frac{2\Gamma(B^+\rightarrow K^+\pi^0)}{\Gamma(B^+\rightarrow K^0\pi^+)} = \left| \frac{p'+t'+c'}{p} \right|^2 = 1.18 \pm 0.12 \]

Possibilities: (1) New physics (e.g., in EWP); (2) $\pi^0$ efficiency underestimated $\pi^0$ efficiency cancels in $(R_nR_c)^{1/2} = 0.95 \pm 0.08$: leads to bound $\gamma \leq 90^\circ$
$B \rightarrow VP$ INFORMATION

More amplitudes than in $B \rightarrow PP$ but data are now abundant

Features of a global flavor SU(3) fit to decays:

Label amps. by meson (pseudoscalar $P$ or vector $V$) with spectator quark

Find $|t_P/t| \simeq f_\rho/f_\pi$ as would be expected for current producing meson

Penguin amplitudes satisfy $p'_V \simeq -p'_P$ as proposed long ago by Lipkin

Small CP asymmetries in many processes imply small strong phases

Time-dependent asymmetries in $B \rightarrow \rho\pi$ resolve discrete ambiguities

Results (see hep-ph/0307395 for details):

Solutions $\gamma = (26 \pm 5)^\circ, (63 \pm 6)^\circ$ (√ CKM fits), $(162_{-5}^{+5})^\circ$.

Solution √ CKM fits $(51^\circ-73^\circ$ at 95% c.l.): small strong phases (√ QCD fact.)

<table>
<thead>
<tr>
<th>As yet unseen decay mode</th>
<th>Predicted $B$ Units of $10^{-6}$</th>
<th>Present limit</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow K^{*0}K^+$</td>
<td>$0.50 \pm 0.05$</td>
<td>$&lt; 5.3$</td>
<td>Pure $p_P$</td>
</tr>
<tr>
<td>$B^+ \rightarrow K^{*+}\pi^0$</td>
<td>$15.0^{+3.3}_{-2.8}$</td>
<td>$&lt; 31$</td>
<td>EWP enhancement</td>
</tr>
<tr>
<td>$B^+ \rightarrow \rho^+K^0$</td>
<td>$12.6 \pm 1.6$</td>
<td>$&lt; 48$</td>
<td>Pure $p'_V$</td>
</tr>
<tr>
<td>$B^0 \rightarrow \rho^0K^0$</td>
<td>$7.2^{+2.1}_{-1.9}$</td>
<td>$&lt; 12.4$</td>
<td>EWP enhancement</td>
</tr>
</tbody>
</table>
\( B \rightarrow VP \) FIT DETAILS

Parameters: \( p'_{P,V} \) (penguin amplitudes); their relative phase \( \phi \)

\( t_{P,V} \) (tree amplitudes); their strong phases \( \delta_{P,V} \) with respect to \( p_{P,V} \)

Color-suppressed \( c_{P,V} \) taken real with respect to \( t_{P,V} \)

Electroweak penguins \( P'_{EW(P,V)} \) taken real with respect to \( p'_{P,V} \)

And finally, the weak phase \( \gamma \)

Total of 12 parameters (11 if \( p'_V/p'_P \) real, 10 if \( p'_V/p'_P = -1 \)); 34 data points

Dashed: \( p'_V/p'_P = -1 \)

Dash-dotted: \( p'_V/p'_P \) real

Solid: \( p'_V/p'_P \) arbitrary

Vertical dashed lines:

CKM fit \( \gamma \) limits
$B \rightarrow VP$ PHASES

Relative amplitude phases specified by both decay rates and CP asymmetries

t_V$ and $t_P$ have small relative phase

$p'_P$ and $p'_V$ add constructively in $B \rightarrow K^*\eta$

Weak phases of $t_{V,P}$ and $\bar{t}_{V,P}$ are included

Constructive $t$-$p$ interference in $B^0 \rightarrow K^{*+}\pi^-$

Destructive interference in $B^0 \rightarrow K^+\rho^-$
$B \to PP$ DECAYS WITH $\eta$, $\eta'$

Large CP asymmetries in $B^+ \to \pi^+ \eta$ imply comparable $A_{CP}$ in $B^+ \to \pi^+ \eta'$

$B \to PP$ analysis by Chiang et al., hep-ph/0306021 $\to$ PR D 68 (2003).

$$A_{CP}(\pi^+ \eta) = -\frac{0.91 \sin \alpha \sin \delta}{1 - 0.91 \cos \alpha \cos \delta} = -0.51 \pm 0.19 \text{ (BaBar)}$$

$$\bar{B}(\pi^+ \eta) = 4.95 \times 10^{-6} (1 - 0.91 \cos \alpha \cos \delta) = (4.12 \pm 0.85) \times 10^{-6}$$

$$A_{CP}(\pi^+ \eta') = -\frac{\sin \alpha \sin \delta}{1 - \cos \alpha \cos \delta} \simeq -0.57 \text{ predicted}$$

$$\bar{B}(\pi^+ \eta') = 3.35 \times 10^{-6} (1 - \cos \alpha \cos \delta) \simeq 2.7 \times 10^{-6} \text{ predicted}$$

Central values:

$$(\alpha, \delta) \simeq (78^\circ, 28^\circ)$$

and $(\alpha \leftrightarrow \delta)$ or

$\alpha \to \pi - \alpha$,

$\delta \to \pi - \delta$

Trees and penguins

of comparable magnitude

in these processes
$B \rightarrow D_{CP}K$ CONSTRAINTS

Update of discussion by M. Gronau, FPCP 2003, hep-ph/0306308

Compare $B^\pm \rightarrow D_{CP}K^\pm$ with (favored) $B^- \rightarrow D^0K^-$

$$\frac{A(\bar{b}\rightarrow u\bar{c}s)}{A(\bar{b}\rightarrow \bar{c}us)} = re^{i(\gamma+\delta)} , \quad \frac{A(b\rightarrow u\bar{c}s)}{A(b\rightarrow c\bar{u}s)} = re^{-i(-\gamma+\delta)}$$

$$R_\pm \equiv \frac{\Gamma(D_{CP}^0=\pm K^-)+\Gamma(D_{CP}^0=\pm K^+)}{\Gamma(D^0K^-)} = 1 + r^2 \pm 2r \cos \gamma \cos \delta$$

$$A_\pm \equiv \frac{\Gamma(D_{CP}^0=\pm K^-)-\Gamma(D_{CP}^0=\pm K^+)}{\Gamma(D_{CP}^0=\pm K^-)+\Gamma(D_{CP}^0=\pm K^+)} = \pm 2r \sin \gamma \sin \delta / R_\pm$$

H. Jawahery (Lepton Photon 2003) for summary of data:

<table>
<thead>
<tr>
<th></th>
<th>$R_+$</th>
<th>$A_+$</th>
<th>$R_-$</th>
<th>$A_-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>1.06 ± 0.31</td>
<td>0.17 ± 0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>1.21 ± 0.29</td>
<td>0.06 ± 0.19</td>
<td>1.41 ± 0.31</td>
<td>−0.19 ± 0.18</td>
</tr>
<tr>
<td>Average</td>
<td>1.14 ± 0.21</td>
<td>0.10 ± 0.15</td>
<td>1.41 ± 0.31</td>
<td>−0.19 ± 0.18</td>
</tr>
</tbody>
</table>

Average $R_\pm \rightarrow 1 + r^2 = 1.227 ± 0.174$ so $r \geq 0.23$ at 1σ

Average $|A_\pm| = 0.140 ± 0.115 \leq 0.255$ at 1σ

Most conservative bound for smallest $r$, largest $|A_\pm|$
$\gamma$ FROM $B \rightarrow D_{CP}K$

One $R$ must be below $1 + r^2$ while other is above it

At $1\sigma$ find $\gamma > 71^\circ$
STATUS, FUTURE – SUMMARY

Status: promising bounds, statistics limited. At 1σ:

\[ R \left( K^+\pi^- \text{ vs. } K^0\pi^+ \right) \text{ gives } \gamma \leq 78° \]
\[ R_c \left( K^+\pi^0 \text{ vs. } K^0\pi^+ \right) \text{ gives } \gamma \geq 44° \]
\[ R_n \left( K^+\pi^- \text{ vs. } K^0\pi^0 \right) \text{ should } = R_c; (R_cR_n)^{1/2} \rightarrow \gamma \leq 90° \]
\[ B \rightarrow DK \text{ decays using } D \rightarrow \text{ CP eigenstates give } \gamma \geq 71° \]

\[ B \rightarrow VP \text{ analysis favors } \gamma = (63 \pm 6)°, 51°–73° \text{ at 95% c.l.} \]

Predict several as yet unseen modes, e.g., \( B^+ \rightarrow \rho^0K^+, K^{*+}\pi^0 \)

SU(3) relations among rate differences remain to be critically tested

Comparable \( B \rightarrow PP \text{ analysis still in progress; } \eta' \text{ predictions} \)

Expect \( 2.0 \leq \bar{B}(\pi^+\eta')/10^{-6} \leq 3.5, -0.34 \leq A_{CP}(\pi^+\eta') \leq -0.80 \)

Global analysis complicated by possible \( B \rightarrow K\pi \) inconsistencies

Future: in experimentalists’ hands until error \( \Delta\gamma < 10° \)

Better error estimates require flavor SU(3) tests at levels of \( B \simeq 1/2 \times 10^{-6} \)

QCD factorization (Beneke-Neubert) promising: small strong phases, symmetry breaking; problems with \( B \rightarrow VP \) penguins