INFLATION FROM WRAPPED BRANES

Sarah Shandera
Columbia University

Can string theory suggest cosmologically interesting ideas?

- or -

Is there a useful discriminating feature of inflation models?

- or -

Could there be *distinctive* signatures of string theory in inflation?

GOALS:

I. Why care? (DBI as a pheno model: non-Gaussianity, tensor/scalar ratio)

II. The wrapped brane inflaton (Relating observables to microphysics and extending the field range)

III. Matching observations with a wrapped brane

IV. Consistency?
I. WHY DBI?
WHAT DO WE WANT FROM INFLATION?

- Enough e-folds (flat potential?)
- Spectrum of primordial fluctuations (amplitude, scale-dependence, correlation functions)
- Other observables?
THE (FIELD THEORY) PICTURE

\[ V(\phi) \]

- **Quantum Fluctuations**
- **Slow-Roll Region**
- **Damped Oscillations, Reheating**
Inflaton $\sim$ Brane separation

- Brane/anti-brane potential from closed string exchange
- Reheating and cosmic strings from brane annihilation

Dvali and Tye; Garcia-Bellido, Rabadan, Zamora; Burgess et al.
THE BIG PICTURE...

3 + 1 dimensions (us)  X  6 compact dimensions

CARTOON GUIDE TO BRANE INFLATION (IN IIB)

Features:
- Mobile D3s
- KS throats
- anti-D3s in throats
- Wrapped D7s

Kallosh; Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi; X. Chen; Dasgupta, Herdeiro, Hirano,
WHY USE THE THROAT?

- Warping helps flatten the brane/anti-brane potential
- Metric is known
- Details of the bulk can be largely ignored
- Warping gives interesting features
THE DBI ACTION

- **Dynamics** (Silverstein, Tong, Alishahiha)

\[ S = - \int d^4 x \ a(t)^3 f(\phi)^{-1} \sqrt{1 - f(\phi) \dot{\phi}^2} + V(\phi) \]

- **Geometry** (Klebanov, Strassler)

\[ ds^2 = h^{-1/2}(r) \eta_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(r) (dr^2 + r^2 ds_{T^{1,1}}^2) \]

\[ f(\phi) = S^{-1} h(\phi) = T_3^{-1} h(\phi) \]
**THE DBI ACTION**

- **Dynamics** (Silverstein, Tong, Alishahiha)

\[
S = - \int d^4x \ a(t)^3 f(\phi)^{-1} \sqrt{1 - f(\phi)\dot{\phi}^2} + V(\phi)
\]

- **Geometry** (Klebanov, Strassler)

\[
ds^2 = h^{-1/2}(r) \eta_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(r) (dr^2 + r^2 ds_{T1,1}^2)
\]

\[
f(\phi) = S^{-1} h(\phi) = T_3^{-1} h(\phi)
\]
THE DBI ACTION

Dynamics (Silverstein, Tong, Alishahiha)

\[ S = - \int d^4 x \ a(t)^3 f(\phi)^{-1} \sqrt{1 - f(\phi) \dot{\phi}^2} + V(\phi) \]

Geometry (Klebanov, Strassler)

\[ ds^2 = h^{-1/2}(r) \eta_{\mu \nu} dx^\mu dx^\nu + h^{1/2}(r) (dr^2 + r^2 ds_{T_{1,1}}^2) \]

[ normalization ]

Assume quadratic

\[ f(\phi) = S^{-1} h(\phi) = T_3^{-1} h(\phi) \]
THE DBI ACTION

Dynamics (Silverstein, Tong, Alishahiha)

\[ S = - \int d^4x \ a(t)^3 \ f(\phi)^{-1} \sqrt{1 - f(\phi)\dot{\phi}^2} + V(\phi) \]

Geometry (Klebanov, Strassler)

\[ ds^2 = h^{-1/2}(r) \eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(r)(dr^2 + r^2 ds_{T_{1,1}}^2) \]

\[ f(\phi) = S^{-1} \ h(\phi) = T_3^{-1} \ h(\phi) \]

Assume quadratic

normalization

D3 brane

From the DBI action, there is an effective speed limit set by the warping:

\[ \dot{\phi}^2 < f(\phi)^{-1} = Sh(\phi)^{-1} \]

\[ h \approx \frac{R^4}{r^4} = \frac{R^4 T_3^2}{\phi^4} \]

Lorentz factor:

\[ \gamma(\phi) = \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}} \]
FRAMEWORK FOR CALCULATING OBSERVABLES

modified Hubble slow roll parameters

\[ \varepsilon_D \equiv \frac{2M_p^2}{\gamma} \left( \frac{H'(\phi)}{H(\phi)} \right)^2 \]

\[ \eta_D \equiv \frac{2M_p^2}{\gamma} \left( \frac{H''(\phi)}{H(\phi)} \right) \]

\[ \kappa_D \equiv \frac{2M_p^2}{\gamma} \left( \frac{H'\gamma'}{H\gamma} \right) \]

\[ \ddot{a} = H^2 (1 - \varepsilon_D) \]

\[ \dot{\phi} = -\frac{2M_p^2 H'}{\gamma} \]

SOME OBSERVABLES

- scalar index
  
  \[ n_s - 1 \approx -4\varepsilon + 2\eta - 2\kappa \]

- tensor index
  
  \[ n_t = \frac{-2\varepsilon}{1 - \varepsilon - \kappa} \]

- tensor/scalar ratio
  
  \[ r = \frac{16\varepsilon}{\gamma} \]


**RELATION BETWEEN DEFINITIONS**

\[ \epsilon_D = -\frac{\dot{H}}{H^2} \rightarrow \epsilon_{SR} \]

\[ \eta_D \rightarrow \eta_{SR} - \epsilon_{SR} \]

\[ \kappa_D = \frac{\dot{c}_s}{c_sH} = s \]

\[ \tilde{\eta} = \frac{\dot{\epsilon}}{\epsilon H} = 2\epsilon_D - 2\eta_D + \kappa \]
CONSTRAINTS

- 55 e-folds in the throat
- Inflation ends when brane separation is small (or expansion parameters become $> 1$)
- Throat is smaller than bulk (volume bound)
- COBE normalization matched

*explore non-Gaussianity, tensor-scalar ratio, etc.*
NON-GAUSSIANNITY
NON-GAUSSIANITY

- Higher order correlations have more information!
- Size (Slow-roll or not? DBI: “$f_{\text{NL}}$” ~ $\gamma^{-2}$)
- Sign (More or less structure? DBI has less)
- Shape
- Scale-dependence

(What kind of physics?)

Carnegie Mellon,
Jan. 16, 2008
THE LOCAL MODEL

- A simple ansatz:
  \[
  \zeta(x) = \zeta_g(x) + f_{NL} [\zeta_g^2(x) - \langle \zeta_g^2(x) \rangle]
  \]

- Then in Fourier space:
  \[
  \langle \zeta_{NG}(k_1) \zeta_{NG}(k_2) \rangle = \langle \zeta_G(k_1) \zeta_G(k_2) \rangle + O(f_{NL}^2)
  \]
  \[
  \approx (2\pi)^3 \delta(k_1 + k_2) \frac{2\pi^2 P_{\zeta_G}(k)}{k^3}
  \]
  \[
  \langle \zeta_{NG}(k_1) \zeta_{NG}(k_2) \zeta_{NG}(k_3) \rangle = f_{NL} \frac{(2\pi)^7}{2} \delta^3(k_1 + k_2 + k_3) \left( \frac{P_{\zeta_G}(k_1) P_{\zeta_G}(k_2)}{k_1^3 k_2^3} + \text{perm.} \right) + O(f_{NL}^3)
  \]
HOW NON-GAUSSIAN?
(SMooth MODELS)

- Slow-roll (squeezed limit): \( f_{NL} \sim -(n_s - 1) \) (Maldacena)

- EFT suggests adding higher derivative terms gives \( f_{NL} \sim 1 \)
  (Creminelli; ‘equilateral model’)

- DBI: Observationally limited (saturates CMB bound)
  (Silverstein, Tong)

  Surprisingly large: the square root sums an infinite number of powers of the derivative; similar effect found in tachyon actions (Barnaby, Cline)
DBI NON-GAUSSIANITY

• DBI is a subset of small sound speed models, 

\[ c_s^2 = \frac{\partial p}{\partial \phi} \frac{\partial \rho}{\partial \dot{\phi}} = \frac{1}{\gamma^2} \]  

(Seery, Lidsey; Chen, Huang, Kachru, Shiu)

• DBI 3-point is largest in the equilateral limit (local is largest in squeezed limit)

• Current CMB bound:  

\[ f_{\text{eff}}^{(k_1 = k_2 = k_3)} \]  

\[ -256 < f_{\text{eq}}^{\text{NL}} < 332 \]  

(Creminelli et al.)

Carnegie Mellon,  
Jan. 16, 2008
Defining an effective $f_{NL}$ from the equilateral limit:

For DBI: \( c_s(k) = c_s^{pivot} \left( \frac{k}{k_{pivot}} \right)^\kappa \)

- For DBI:
  \[
f_{NL}^e = -\frac{35}{108} \frac{3^{2(n_s-1)}}{c_s^2 - 1}
\]

- We can use the DBI case to suggest an ansatz, with $\kappa$ a free parameter

\[
f_{NL}^{eq}(k) = f_{NL, pivot}^{eq} \left( \frac{k}{k_{pivot}} \right)^{-2\kappa}
\]
THE CMB IS ONLY THE BEGINNING...

(WMAP 3, Spergel et al.)
RELEVANT OBSERVATIONS

\[ \kappa = -0.3 \]

\[ \kappa = -0.1 \]
Phyically (local model)... 

Negatively skewed $f_{NL} = \pm 0.1$ 
Positively skewed $\sigma^2 = 1$ 

$k^2 \zeta = 4\pi G\alpha^2 \delta \rho$ 
$+ f_{NL} =$ more clusters
OBSERVABLES

- CMB
- Cluster number counts
- Galaxy bispectrum

Analysis on the next slide from arXiv:0711.4126, M. LoVerde, A. Miller, L. Verde, S.S.
SO HOW WELL CAN WE DO?

<table>
<thead>
<tr>
<th>Info.</th>
<th>Fiducial Model</th>
<th>( \sigma_{\Omega_m} )</th>
<th>( \sigma_h )</th>
<th>( \sigma_{\sigma_8} )</th>
<th>( \sigma_{f_{NL}} )</th>
<th>( \sigma_{\kappa} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP</td>
<td>( f_{NL}^{eq} = 38, \kappa = 0 )</td>
<td>0.0264</td>
<td>0.029</td>
<td>0.046</td>
<td>150</td>
<td>–</td>
</tr>
<tr>
<td>WMAP + ( dN/dz )</td>
<td>( f_{NL}^{eq} = 38, \kappa = 0 )</td>
<td>0.0080</td>
<td>0.029</td>
<td>0.026</td>
<td>150</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 38, \kappa = -0.3 )</td>
<td>0.011</td>
<td>0.029</td>
<td>0.032</td>
<td>150</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = -256, \kappa = 0 )</td>
<td>0.0076</td>
<td>0.029</td>
<td>0.022</td>
<td>150</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = -256, \kappa = -0.3 )</td>
<td>0.0089</td>
<td>0.029</td>
<td>0.022</td>
<td>149</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 332, \kappa = 0 )</td>
<td>0.010</td>
<td>0.029</td>
<td>0.034</td>
<td>150</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 332, \kappa = -0.3 )</td>
<td>0.011</td>
<td>0.029</td>
<td>0.034</td>
<td>150</td>
<td>0.23</td>
</tr>
<tr>
<td>Planck</td>
<td>( f_{NL}^{eq} = 38, \kappa = 0.0 )</td>
<td>0.0084</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>–</td>
</tr>
<tr>
<td>Planck + ( dN/dz )</td>
<td>( f_{NL}^{eq} = 38, \kappa = -0.3 )</td>
<td>0.0058</td>
<td>0.011</td>
<td>0.014</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = -256, \kappa = 0 )</td>
<td>0.0070</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = -256, \kappa = -0.3 )</td>
<td>0.0053</td>
<td>0.011</td>
<td>0.013</td>
<td>40</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 332, \kappa = 0 )</td>
<td>0.0061</td>
<td>0.011</td>
<td>0.013</td>
<td>40</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 332, \kappa = -0.3 )</td>
<td>0.0066</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>( f_{NL}^{eq} = 332, \kappa = -0.3 )</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>0.11</td>
</tr>
</tbody>
</table>

\( \Omega_m = 0.24, h = 0.73, \sigma_8 = 0.77, \kappa = 0 \)

SO HOW WELL CAN WE DO?

<table>
<thead>
<tr>
<th>Info.</th>
<th>Fiducial Model</th>
<th>$\sigma_{\Omega_m}$</th>
<th>$\sigma_h$</th>
<th>$\sigma_{\sigma_8}$</th>
<th>$\sigma_{f_{NL}}$</th>
<th>$\sigma_{\kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP</td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td>WMAP + $dN/dz$</td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = -256 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = -256 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 332 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 332 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td>Planck</td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td>Planck + $dN/dz$</td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 38 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = -256 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = -256 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 332 \quad \kappa = 0$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_{NL}^{eq} = 332 \quad \kappa = -0.3$</td>
<td>0.0068</td>
<td>0.011</td>
<td>0.015</td>
<td>40</td>
<td>1.00</td>
</tr>
</tbody>
</table>

($\Omega_m = 0.24, h = 0.73, \sigma_8 = 0.77, \kappa = 0$)
ASPECTS NG MAY TEST

- UV (NG increases on small scales) vs. IR (NG decreases on small scales)
- Deformed conifold: $\kappa = 0$
- Features (bumps in the potential or the warp factor)
- Deviations from Bunch-Davies? (Holman, Tolley)

* All scale-dependent
Here $\kappa$ is a slow-roll parameter; can consider larger $|\kappa|$ if spectrum is computed numerically
Recent analysis finds a NG signal in CMB (Yadav, Wandelt, $f_{NL} \sim 90$)

No matter the eventual fate of that result, NG is such a powerful discriminator that it should continue to be probed at all possible scales
FIELD RANGE AND OBSERVABLES
Field range is related to tensor/scalar ratio:

\[
\frac{1}{M_p} \frac{d\phi}{dN_e} = \sqrt{\frac{r}{8}}
\]

Sub-planckian field range implies \( r < 0.01 \) (barely detectable) if \( r \) is constant.

Can be larger if \( r \) changes rapidly (e.g., if sound speed decreases).
REMINDER: CHAOTIC INFLATION

- Usually, quadratic potential requires trans-Planckian field range

\[ V(\phi) = \frac{m^2 \phi^2}{2} \]

\[ H(\phi) = h_n \phi^n \]

\[ \varepsilon < 1 \Rightarrow \frac{\phi}{M_p} > \sqrt{2} \]

\[ \varepsilon = \frac{2M_p^2}{\gamma} \left( \frac{H'}{H} \right)^2 < 1 \Rightarrow \frac{\phi}{M_p} > n \sqrt{\frac{2}{\gamma}} \]
FUNDAMENTAL QUANTITIES AND OBSERVABLES

- Field Range: canonical inflaton, 4D Planck mass:
  \[
  \frac{\phi}{M_p} = \sqrt{S \rho}
  \]

- Relating the modulus field to the canonical inflaton:

- Volume and Planck mass:
  \[
  M_p^2 = \frac{V^w_6}{\kappa_{10}^2} = \frac{2V^w_6}{(2\pi)^7 g_s^2 \alpha'^4}
  \]
THROAT DETAILS

- $AdS_5 \times X_5$
- Background D3 charge $N$
- $R^4 = \frac{4\pi g_s N \pi^3 \alpha'^2}{\nu}$
- $\nu = Vol(X_5)$
- Cut-off scale $\rho_0$
The smallest possible compact volume is the throat volume:

\[ V_6^w \sim V_6^{throat} = \int_0^{\rho_{UV}} d\rho \, \rho^5 h(\rho) \int d\Omega_{X_5} \]

\[ = 2\pi^4 g_s N^2 \rho_{UV}^2 \]

Then, the field range is:

\[ \left( \frac{\Delta \phi}{M_p} \right)_{\text{max}} < \frac{2}{\sqrt{N}} \]
CONFLICT WITH OBSERVATION?

- Non-Gaussianity and the tensor/scalar ratio:
  \[ \left( \frac{\Delta \phi}{M_p} \right)^2 = \frac{32}{r \gamma^2} \]
  \[ N \lesssim 38 \]

- COBE normalization:
  \[ N \gtrsim 10^8 \text{Vol}(X_5) \]

- Need small background charge, large orbifolding:
  \[ \text{Vol}(X_5) \sim (\pi)^3 \rightarrow 10^{-7} \]

(Baumann, McAllister)
SUMMARY SO FAR...

- DBI brane inflation suggests an alternative to requiring a flat potential; worthwhile to try to realize DBI inflation
- This comes along with large, scale-dependent non-Gaussianity and maybe observable gravitational waves
- DBI with a D3 brane struggles(!) to match data
- Regardless of the viability of DBI, suggests interesting ways to distinguish inflation scenarios
II. THE WRAPPED BRANE INFLATON
In the UV (away from the tip)

In the IR, $S^2$ shrinks to zero size; $S^3$ finite volume

warp factor approaches a constant:

$$h(r_0) = \frac{R^4}{r_0^4} = e^{8\pi K/(3g_sM)}$$
WRAPPING A D5-BRANE

(Kobayashi, Mukohyama, Kinoshita; Becker, Leblond, S.S.)

EXTENDING THE FIELD RANGE

- Orbifolding $S^2$ or $S^3$:
  \[
  \int_{S^2} d\Omega_2 \rightarrow \frac{1}{a} \int_{S^2} d\Omega_2
  \]

- Wrapping number $p$

- Then the normalization is
  \[
  S = \frac{4\pi R^2}{3} \frac{p}{a} T_5
  \]

- Field range:
  \[
  \left( \frac{\Delta \phi}{M_p} \right)^2 \leq \frac{2^3 \pi p}{3} a \left( \frac{g_s}{N v} \right)
  \]
III. MATCHING OBSERVATIONS
THREE DIMENSIONLESS VARIABLES

- Inflaton mass: \[ A \equiv H' = \frac{m}{\sqrt{6M_p}} \]

- Field range: \[ B \equiv \frac{\phi}{M_p} \]

- Normalization: \[ C \equiv R^4S \]
**OBSERVATIONAL CONSTRAINTS**

- **Power spectrum normalization:**
  
  \[ P_S = \frac{H^4 \gamma^2}{16\pi^2 M_p^4 H'^2} = \frac{A^4 C}{4\pi^2} \sim 2 \times 10^{-9} \]

- **Non-Gaussianity:**
  
  \[ |f_{NL}^{eq}| < 256 \quad \gamma \sim 2M_p^2 f(\phi)^{1/2} H' = \frac{2\sqrt{CA}}{B^2} < 30 \]

- **Tensor/Scalar ratio:**
  
  \[ r = \frac{32M_p^2}{\gamma^2 \phi^2} = \frac{8B^2}{CA^2} < 0.3 \]

CONSTRAINTS ON FUNDAMENTAL PARAMETERS

\[ \frac{N}{v} > 10^5 g_s^{-1/3} \left( \frac{a}{p} \right)^{2/3} \]
\[ Nv < 6 \times 10^3 \left( \frac{p}{a} \right)^2 g_s \]

Example point: \( N \sim 10^4, v=1/40; \gamma=25, r=0.29 \)
LYTH BOUND REVISITED

• Observationally, Lidsey and Huston find

\[ r > 0.002 \]

• For a D3, the result was

\[ r < 10^{-7} \]

• Assuming that \( r \) is constant on the scale of roughly 1 e-fold, the Lyth bound with a wrapped D5 gives

\[ r < 0.04 \]
CLUES FROM THE D3 CASE?

- Lots of caveats:
  - Haven’t computed number of e-folds
  - Haven’t considered realistic throat geometry
  - Monte-Carlo for D3 brane case, *without* imposing volume constraint (there are other ways to extend the field range) *(hep-th/0702107, R. Bean, S.S., H. Tye, J. Xu)*
IV. CONSISTENCY ISSUES
SOME POINTS OF CONCERN

- What is the potential really?
- Back-reaction of the brane on the geometry?
- Is this a good “low” energy description?
SOME POSSIBLE POTENTIALS

- Chern-Simons term:
  \[ S_{CS} = \mu_5 \int P[C_6 + C_4(B_2 + 2\pi F)] \]
  \[ = \mu_5 2\pi \int d^4x \left( \frac{3M}{4h} + \frac{1}{g_s h} (3g_s M \ln(\phi/\phi_0) + 2\pi q) \right) \]
  \[ H(\phi) = h_n \phi^n \]

- Other power laws?
  - \( n=2 \) inflates for smaller field range
  - \( n>3 \) require trans-planckian field range
SIMPLISTIC BACKREACTION

• Examining the perturbation to the radial warp factor due to the brane:

\[ p\gamma \ll \sqrt{Nv / gs} \]

• With bounds from data, this gives:

\[ \gamma \ll \sqrt{Nv a / gs \ p} \ll 10^{3/2} \]
Examining the perturbation to the radial warp factor due to the brane:

\[
\frac{p\gamma}{a} \ll \sqrt{\frac{Nv}{g_s}}
\]

With bounds from data, this gives:

\[
\gamma \ll \left(\frac{Nv a}{g_s p}\right)^{1/2} \ll 10^{3/2}
\]

And for D3:

\[
\gamma T_3 < NT_3
\]

\[
\Rightarrow \gamma \ll 36
\]
KK MODES

- We would like to ignore all fields except the inflaton:
  \[ m_{KK}^w > H \]

- KK modes at the bottom of the throat have warped masses
  \[
  \frac{1}{R} \frac{\phi_{IR}}{\phi_{UV}} > H_{\text{max}} = A\phi_{\text{max}}
  \]
  \[
  h_{IR}^{-1/4} > A\sqrt{SR^2} = A\sqrt{C}
  \]

- Number of e-folds?
  \[ N_e = A\sqrt{C}\log(h_{IR}^{1/4}) \]
CONCLUSIONS

- Fundamental moduli space translates to constraints on canonical inflaton field range (matches expectations from EFT)
- For brane inflation, the relationship depends on the dimensionality
- DBI with a wrapped brane can match observational data in the most optimistic case, but:
  - Many concerns other than field range
SOME FINAL OPTIMISM...

- DBI inflation has suggested an alternative to standard slow-roll with observable features
- Non-Gaussianity is especially useful and leads to new ideas for observationally distinguishing models
- DBI emphasizes importance of moduli space
- Inflation is a useful context to investigate warped supersymmetry breaking
\[
\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = (2\pi)^7 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \frac{\mathcal{P}_\zeta(K)^2}{k_1^3 k_2^3 k_3^3} (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s)
\]

\[K = k_1 + k_2 + k_3\]

(Chen, Huang, Kachru, Shiu)

Current CMB bound: \(f_{NL}^{eff} (k_1 = k_2 = k_3)\)

\[-256 < f_{NL}^{eq} < 332\] (Creminelli et al.)
\[
\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{\mathcal{P}_\zeta(K)^2}{k_1^3 k_2^3 k_3^3} (A_\lambda + A_c + A_o + A_\epsilon + A_\eta + A_s)
\]

Current CMB bound: \[ f_{NL}^{eff}(k_1 = k_2 = k_3) \]

\[ -256 < f_{NL}^{eq} < 332 \] (Creminelli et al.)
GENERAL SOUND SPEED

\[
\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = (2\pi)^7 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \frac{P^\zeta(K)^2}{k_1^3 k_2^3 k_3^3} (A_\lambda + A_c + A_o + A_\epsilon + A_\eta + A_s)
\]

\[K = k_1 + k_2 + k_3\]

\[\sim 0 \text{ for DBI}\]

\[\text{lower order}\]

\[A^c(k_1, k_2, k_3) = \left( \frac{1}{c_s^2} - 1 \right) \left( -\frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i\neq j} k_i^2 k_j^2 + \frac{1}{8} \sum_i k_i^3 \right)\]

(Chen, Huang, Kachru, Shiu)

Current CMB bound: \[f_{NL}^{eff}(k_1 = k_2 = k_3)\]

\[-256 < f_{NL}^{eq} < 332\] (Creminelli et al.)
TWO COMPARISONS

\[
\frac{A_{\text{local}} - A_c}{A_c}
\]

\[
\frac{A_{\text{equil}} - A_c}{A_c}
\]

Carnegie Mellon,
Jan. 16, 2008