$V_{cb}$: experimental and theoretical highlights

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The method

- Ultimate goal: a precise determination of $V_{cb}$
- The challenge: precise evaluation of the hadronic matrix element
The exclusive approach: HQET & $V_{cb}$

- **Heavy Quark Effective THEORY (HQET) (Isgur & Wise)**
  - QCD is flavor independent, so in the limit of infinitely heavy quarks $q_a \to q_b$ occurs with unit form-factor $[F(1)=1]$ when the quarks are moving with the same invariant 4-velocity, $w=1$.
  - **Example:** for $B \to D^* \ell \nu$:
    - All form-factors are related to one universal shape that can be measured
    - Corrections to F(1) due to finite quark masses are calculable along with QCD corrections. These corrections are parameterized in a series: $\sum_n C_n (1/m_{q_i})^n$, $n=1, 2...$
\( V_{cb} \) from \( B \rightarrow D^* \ell \nu \\

- **HQET:**
  \[
  \frac{d\Gamma}{dw} = K(w)F^2(w)|V_{cb}|^2
  \]
  \[
  F(w) = F_{D^*}(1)g(w)
  \]

- The shape, \( g(w) \) not a clearly predictable quantity, but is constrained by theoretical bounds and measured form factors
- Experiments can measure \( d\Gamma/dw \)
- To find \( V_{cb} \) measure value of decay rate at \( w=1 \rightarrow F(1)|V_{cb}| \)
$F(1)|V_{cb}|$ using $B \to D^{*}\ell\nu$

- Fit to function shape given by Caprini et al.
- Yields value of $F(1)|V_{cb}|$ & shape, parameterized by $\rho^2$.
- $F(1)|V_{cb}| = (36.7 \pm 0.8) \times 10^{-3}$ (HFAG)
- $\rho^2 = 1.44 \pm 0.14$ (HFAG)
Theoretical calculations of F(1)

• $F(1) = \eta_{\text{QED}} \cdot \eta_{\text{QCD}} \left(1 + \delta_{1/m^2} + \ldots\right)$
  - Lukes theorem: no $\delta_{1/m}$ corrections (would be in $D(\nu)$)
  - $\eta_{\text{QED}} = 1.007$, $\eta_{\text{QCD}} = 0.960 \pm 0.007$ at two loops
  - $\delta_{1/m^2}$ involves $1/m_b^2$, $1/m_c^2$, $1/m_c m_b$

• First Lattice Gauge calculations (quenched-no light quark loops) $0.913^{+0.024+0.017}_{-0.017-0.030}$
  ultimate solution

• PDG (Artuso & Barberio) $F(1) = 0.91 \pm 0.05$
$V_{cb}$ Exclusive Averages

\[
V_{cb}\text{ excl} = (40.03 \pm 0.9_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}
\]
Another exclusive channel: $B \to D\ell\nu$

- Renewed interest on this channel:
  - Lattice calculations
  - QCD sum rules evaluation of $G(1)$
- Using $G(1) = 1.058 \pm 0.07$ (Artuso-Barberio PDG2002)

$V_{cb} = (39.8 \pm 3.5_{\text{exp}} \pm 2.9_{\text{th}}) \times 10^{-3}$
\[ |V_{cb}| \text{ from inclusive } B \rightarrow X_c \ell \nu \]

- From \( B(B \rightarrow X_c \ell \nu) \) extract the experimental decay width:
  \[ \Gamma_{sl}^c \equiv \frac{B(b \rightarrow X_c l\nu)}{\tau_b} \]

- Compare with the theoretical prediction from Operator Product Expansion:
  \[
  \Gamma_{sl}^c = \frac{G_F^2 m_b^5 |V_{cb}|^2}{192\pi^3} \left[ z_0 \left( 1 - \frac{\mu_{\pi}^2 - \mu_G^2}{2m_b^2} \right) - 2 \left( 1 - \frac{m_c^2}{m_b^2} \right) \frac{\mu_G^2}{m_b^2} - \frac{2\alpha_s}{3\pi} z_0^{(1)} + \ldots \right]
  \]

Known phase space factors
The Heavy Quark Expansion

- Theoretical framework: Heavy Quark Expansion:
  - Inclusive properties expressed as asymptotic expansion in terms of the “energy release” $m_b-m_c$
  - Underlying theoretical accuracy: are all the uncertainties quantified? In particular ansatz of quark-hadron duality.
  - Experimental determination of the Heavy quark expansion parameters, in particular:
    - $m_b, m_c$ at the relevant mass scale
    - $\mu_\pi^2 [\lambda_1]$ kinetic energy of the b quark
    - $\mu_G^2 [\lambda_2]$ expectation value of chromomagnetic op.
**m_b: a multifaceted fundamental parameter**

<table>
<thead>
<tr>
<th>Method</th>
<th>( m_{\text{kin}} ) (GeV)</th>
<th>( \overline{m}_b(\overline{m}_b) ) (GeV)</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beneke, Signer, Smirnov</td>
<td>-</td>
<td>4.26±0.12</td>
<td>Sum rules</td>
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<td>Melnikov</td>
<td>4.56±0.06</td>
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<td>Hoang</td>
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<tr>
<td>Jamin, Pich</td>
<td>-</td>
<td>4.19±0.06</td>
<td>Sum rules, no resummation</td>
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<td>Pineda, Yndurain</td>
<td>-</td>
<td>4.44±0.04</td>
<td>Q(1S) mass</td>
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<tr>
<td>NRQCD</td>
<td>-</td>
<td>4.28±0.03±0.03±0.10</td>
<td>Lattice HQET (( n_f=2 ))</td>
</tr>
</tbody>
</table>

Important for \( V_{c(u)b} \)

Y expansion

Jet observables sensitive to \( b \) mass (LEP)

\(+\) pole mass \( m_{b,\text{pole}} \approx m_{\text{kin}} +0.255 \) GeV Bigi-Mannel hep/ph/0212021
Problems with HQE

- Terms in $1/m_b^3$ are multiplied by unknown functions; hard to evaluate error due to these higher order terms
- Duality is assumed: integrated over enough phase space the exclusive charm bound states & the inclusive hadronic result will match at quark-level. But no way to evaluate the error...
- Appears to miss $\Lambda_b$ lifetime by $10\pm5\%$ & b-baryon by $18\pm3\%$; however semileptonic decay may be easier
- Need experimental tests to evaluate errors
  - Sharpen our knowledge of B meson semileptonic decays with high $M_x$ hadronic states
  - Perhaps use $V_{cb}$ as a test?
  - ...
How to Measure $\lambda_1$ & $\overline{\Lambda}$

- Can determine $\lambda_1$ and $\overline{\Lambda}$, and thus $V_{cb}$ by measuring “moments” in semileptonic decays
  - Hadronic mass moments (ex: $\langle M_X^2 - M_D^2 \rangle$, $M_D$ is spin-averaged $D, D^*$ mass) where $B \rightarrow X \ell \nu$
  - Semileptonic moments

- Can also use $b \rightarrow s \gamma$ decays, here we use the 1$^{st}$ moment of the photon energy
• Hadronic Mass & Lepton Energy moments found in semileptonic decays “detecting the neutrino” using missing energy
• $b \rightarrow s \gamma$ moment determination shown later
• Fitting this & other data Bauer, Ligeti, Luke Manohar find $V_{cb}=(40.8 \pm 0.9) \times 10^{-3}$ & $m_b=4.74 \pm 0.10$ GeV (hep-ph/0210027)

\[ \Lambda=0.35 \pm 0.07 \text{ GeV} \]
\[ \lambda_1 = -0.24 \pm 0.07 \text{ GeV}^2 \]
exp errors only
BaBar Moments Result

- Using only BaBar hadronic moments & $B_s$:
- $V_{cb} = (42.1 \pm 1.0 \pm 0.7) \times 10^{-3}$ again within $\pm 7\%$ of $D^* \ell \nu$
- $m_b^{1S} = 4.64 \pm 0.09 \pm 0.09$ GeV
- $(M_x^2$ as function of lepton momentum, is now consistent with theory)
Comparison of Hadron & Lepton Moments (BaBar)

- Lepton & Hadron moments differ somewhat. Does this indicate a Duality violation?
- Difference of 0.2 GeV in $m_b$ leads to 20% difference in $V_{ub}$
Refined experimental results agree with theory.

Can we draw any definitive conclusion?

\[ M_X^2 = 0.534 \pm 0.041 \pm 0.074 \]
Summary of experimental results

• $V_{cb}^{\text{excl}} = (40.03 \pm 0.9_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}$

• $V_{cb}^{\text{incl}} = (41.5 \pm 0.4_{\text{Is}} \pm 0.4_{\lambda} \pm 0.4_{\Lambda_{\text{meas}}} \pm 0.9_{\text{th}}) \times 10^{-3}$

Future prospects:

• Precise form factor calculations from lattice gauge calculation

• More extensive exploration of inclusive semileptonic decay observables: in particular high $M_x$ component

• More detailed evaluation & validation of theoretical errors