Notes on $|V_{ub}|$

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Thanks to
D. Cronin-Hennessy
T. Meyer
E. Thorndike

$|V_{ub}|$

How do we assign noncontroversial errors?!

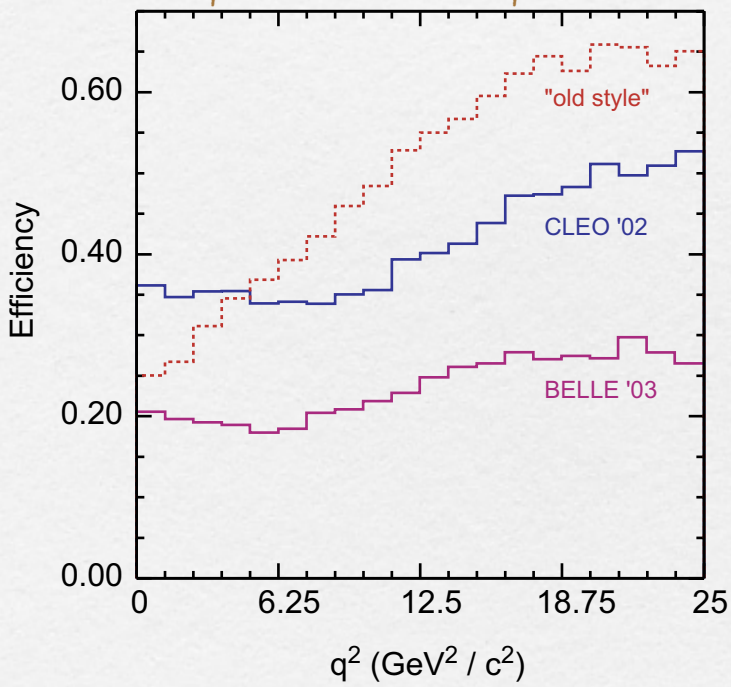
Overview

- 10 years of $|V_{ub}|$
  - marked improvement in understanding

Akhoury, Ball, Bauer, Bigi, De Fazio, Ligeti, Luke, Mannel, Manohar, Neubert, Rothstein, Ruckl, Shifman, Uraltsev, Vainshtein, Voloshin, Wise, + lattice community + ...

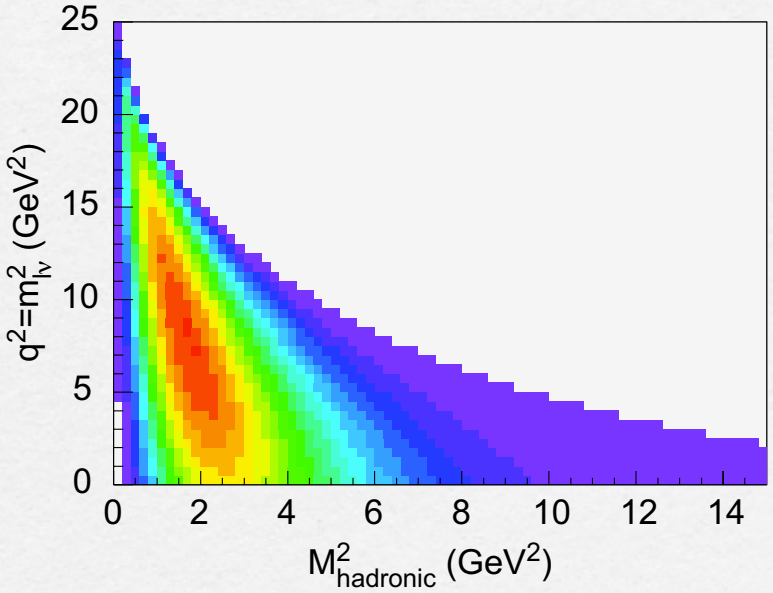
- theory: improvements + categorization of unknown corrections
- experiment: rates without strong reliance on (unknown) theory

"endpoint": model dependence



We can now limit largest unknown uncertainties

Inclusive $|V_{ub}|$



Operator Product Expansion (OPE)

$$\Gamma(B \to X_u e\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \times \left[1 - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots - \mathcal{O}(\frac{\alpha_s}{\pi})\right]$$

b-spin/brown muck hyperfine

b time dilation

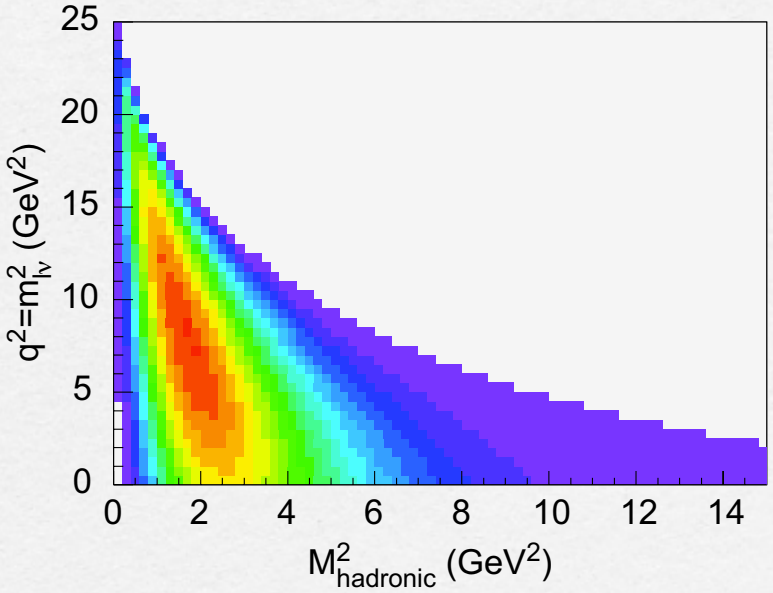
Complete to α_s^2

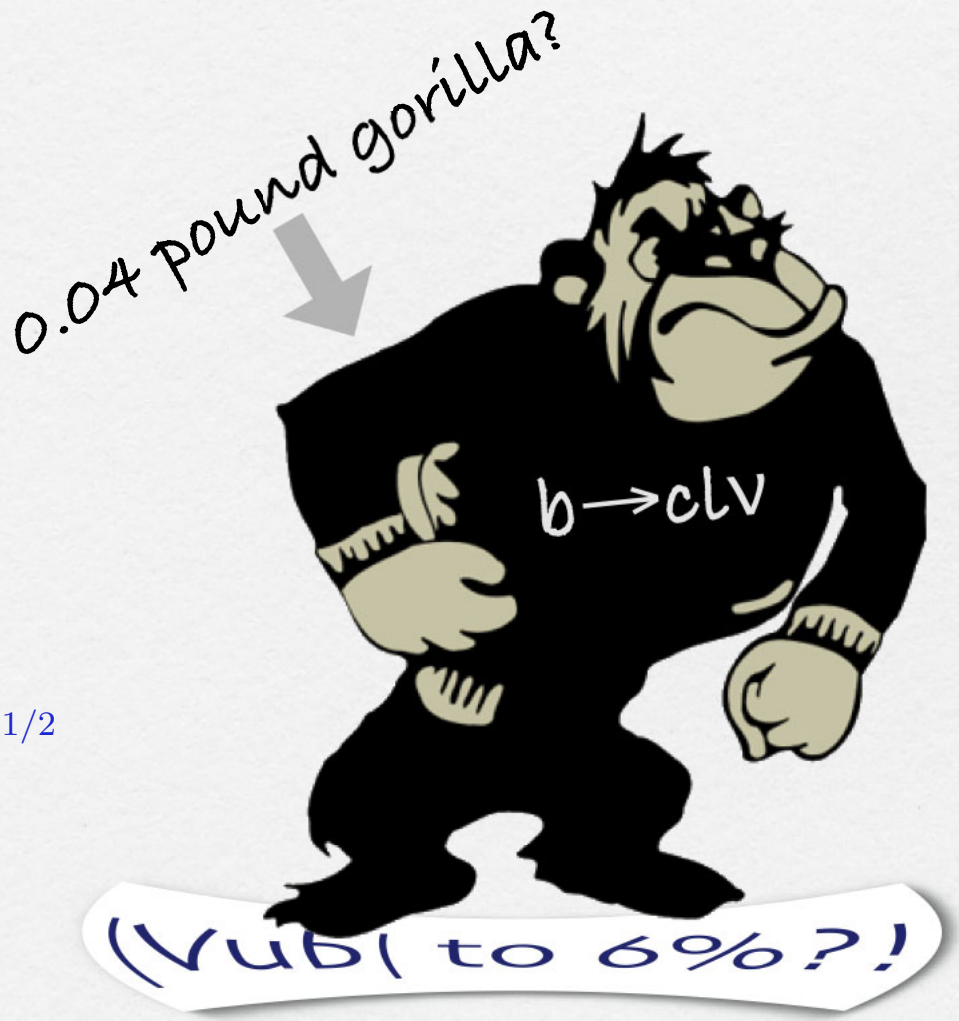
- Issues:
  - Global quark hadron duality?
    - |Vcb| excl. vs. incl. OK (few %)
  - dominant uncertainty:
    - $m_b^{1S}(1GeV) = 4.58 \pm 0.09$ GeV

$$|V_{ub}| = 0.00445 \times \left(\frac{\mathrm{B}(b \to u\ell\bar{\nu})}{0.002}\frac{1.55\mathrm{ps}}{\tau_b}\right)^{1/2} \times (1 \pm 0.020_{OPE} \pm 0.052_{m_b})$$

|Vub| to 6%?!

Inclusive $|V_{ub}|$





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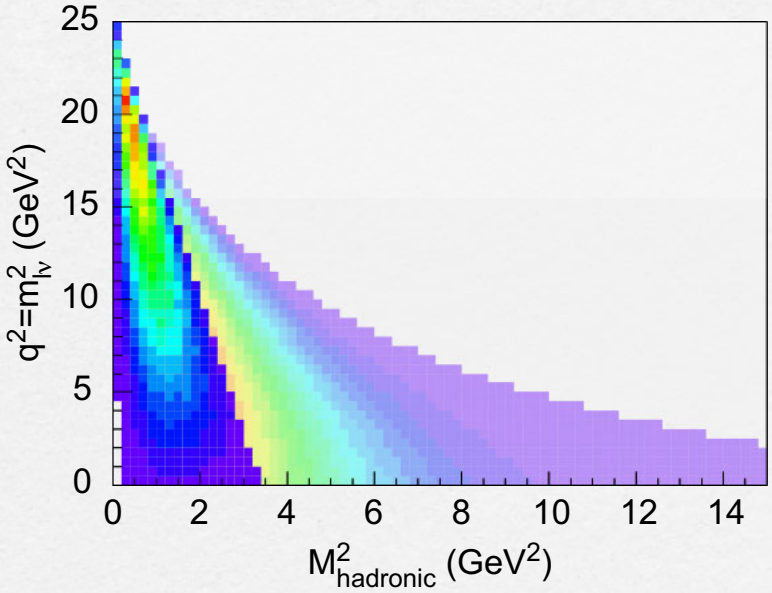
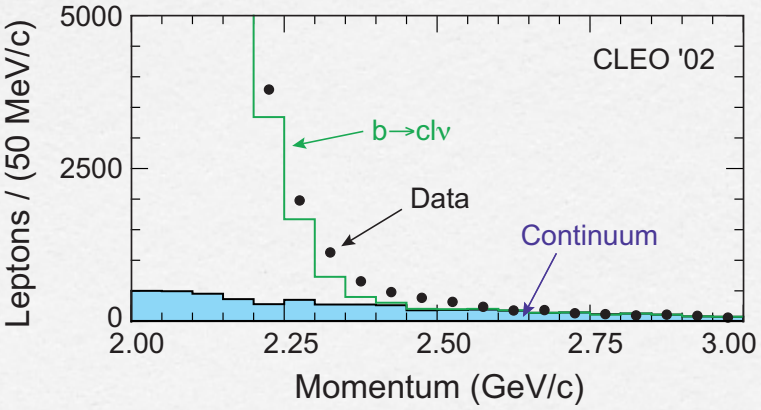
Inclusive: p_l endpoint





- Notes:
  - Measured rates integrated in $\Upsilon(4s)$ frame (to date)
  - Must know fraction of $b \to u l \nu$ events in endpoint region (f_ϵ)
    - Operator Product Expansion breaks down:

$$\frac{E_X \Lambda_{QCD}}{m_x^2} \sim 1$$

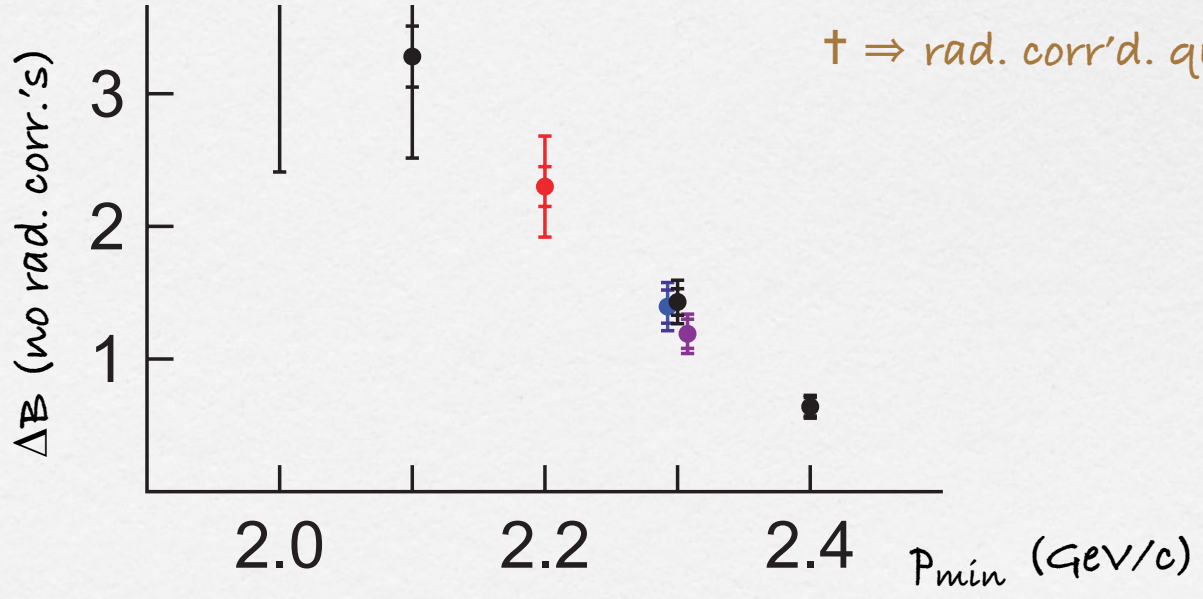
How to get f_ϵ?!

    - radiative (QED) corrections

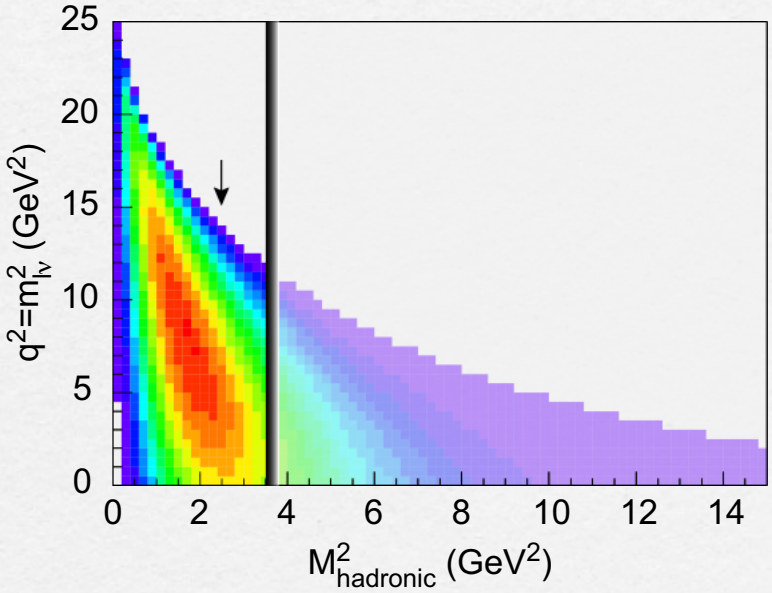
Inclusive: p_l endpoint

p (GeV/c)	$\Delta\mathcal{B}_u(p)(10^{-4})$	f_E	
2.0-2.6	$4.22 \pm 0.33 \pm 1.78$	$\dagger 0.266 \pm 0.041 \pm 0.024$	CLEO (e, μ)
2.1-2.6	$3.28 \pm 0.23 \pm 0.73$	$\dagger 0.198 \pm 0.035 \pm 0.020$	CLEO (e, μ)
2.2-2.6	$2.30 \pm 0.15 \pm 0.35$	$\dagger 0.130 \pm 0.024 \pm 0.015$	CLEO (e, μ)
2.3-2.6	$1.43 \pm 0.10 \pm 0.13$	$0.074 \pm 0.014 \pm 0.009$	CLEO (e, μ)
	$\dagger 1.52 \pm 0.14 \pm 0.14$	$0.078 \pm 0.015 \pm 0.009$	BaBar (e)
	$1.19 \pm 0.11 \pm 0.10$	$\dagger 0.072 \pm 0.014 \pm 0.008$	BELLE (e)
2.4-2.6	$0.64 \pm 0.07 \pm 0.05$	$\dagger 0.037 \pm 0.007 \pm 0.003$	CLEO (e, μ)

† ⇒ rad. corr'd. quantity

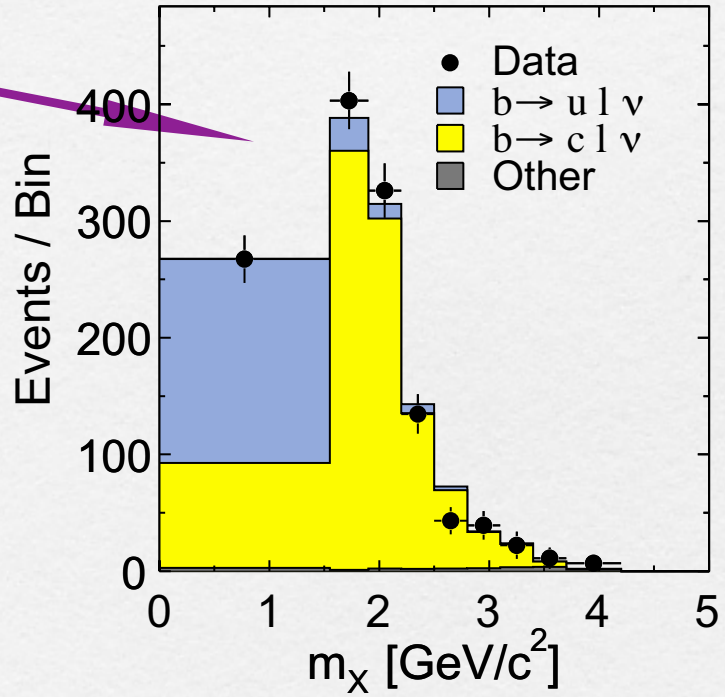


Inclusive: restrict M_X



BaBar

- fully recon. B tag
- p_l>1.0 GeV/c
- M_X<1.55 GeV



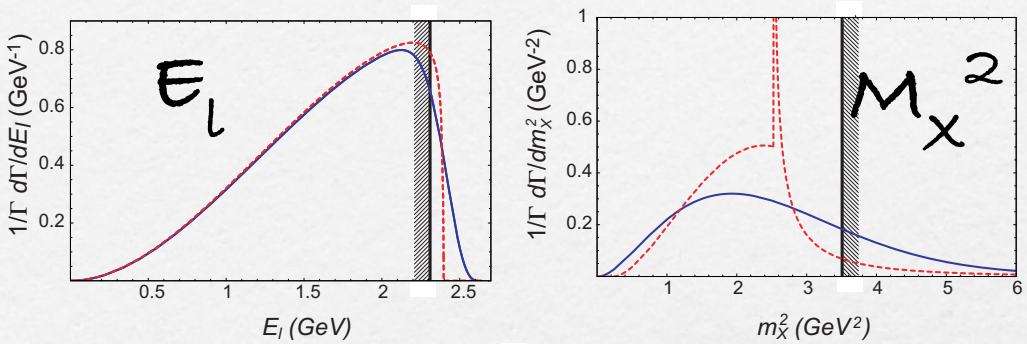
- B-factories: excellent S/B
  - efficiency flat in signal region?
- OPE: again $\frac{E_X \Lambda_{QCD}}{m_x^2} \sim 1$

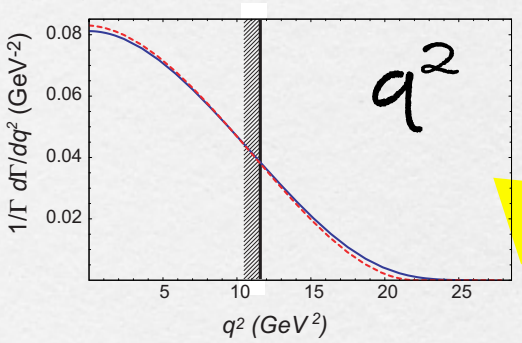
BELLE, too (prelim)

- $D^{(*)}l\nu$ tag (double semileptonic event)
- p_l>1.0 GeV/c
- M_X<1.5 GeV
- not as clean as full reconstruction tag

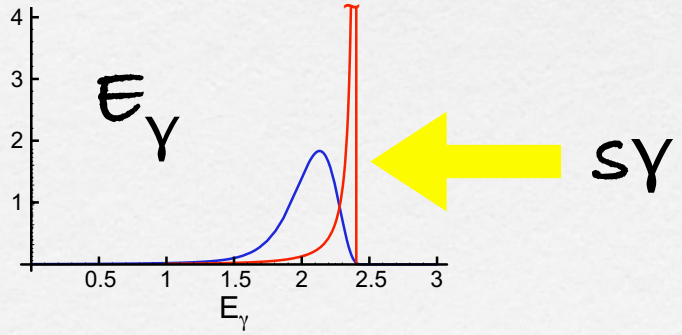
Inclusive: shape func'n

- M_X, p_l rate in restricted regions?
  - re-sum OPE in $\frac{E_X \Lambda_{QCD}}{m_x^2}$
  - `shape function' $f(k_+)$ — $p_b^\mu - m_b v^\mu$ along $m_b v^\mu - q^\mu$
  - $d\Gamma = \int dk_+ \, f(k_+) d\Gamma^{(parton)}_{m_b \to m_b + k_+}$
    - leading order (`twist')
    - universal for b→light quark





ulv



sγ

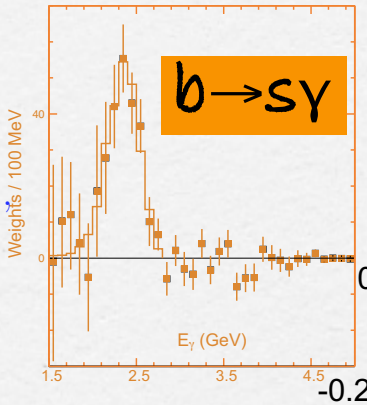
Inclusive: shape func'n

- Ideally:

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi} K_{\rm pert} \frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + O(\Lambda_{\rm QCD}/M_B)$$

weighted $d\Gamma/dE_l$, $d\Gamma/dE_\gamma$

integrals for $b\to u l\nu$, $b\to s\gamma$



+ $F[\Lambda^{SF}, \lambda_1^{SF}](k_+)$

- Currently:
  - efficiency variations
  - E_l: $\Upsilon(4S)$ vs B rest frame
  - no calc's for $\Gamma_u(E_0, M_{X0}, q^2_0)$!

⇒ model shape fcn
(introduces model dep.)

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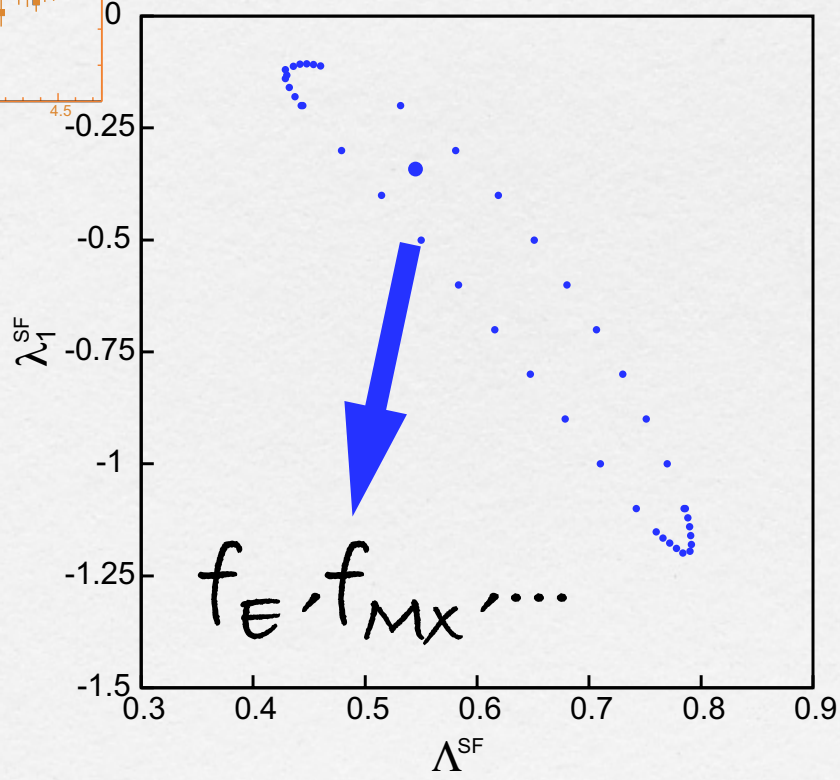

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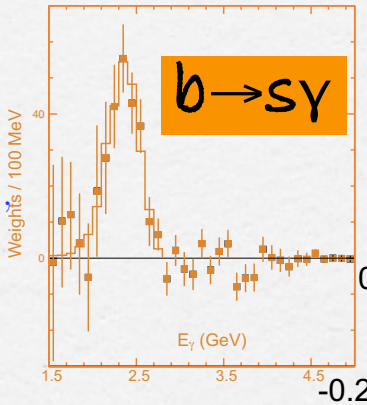
Weighted $d\Gamma/dE_l$, $d\Gamma/dE_\gamma$ integrals for $b\to u l\nu$, $b\to s\gamma$



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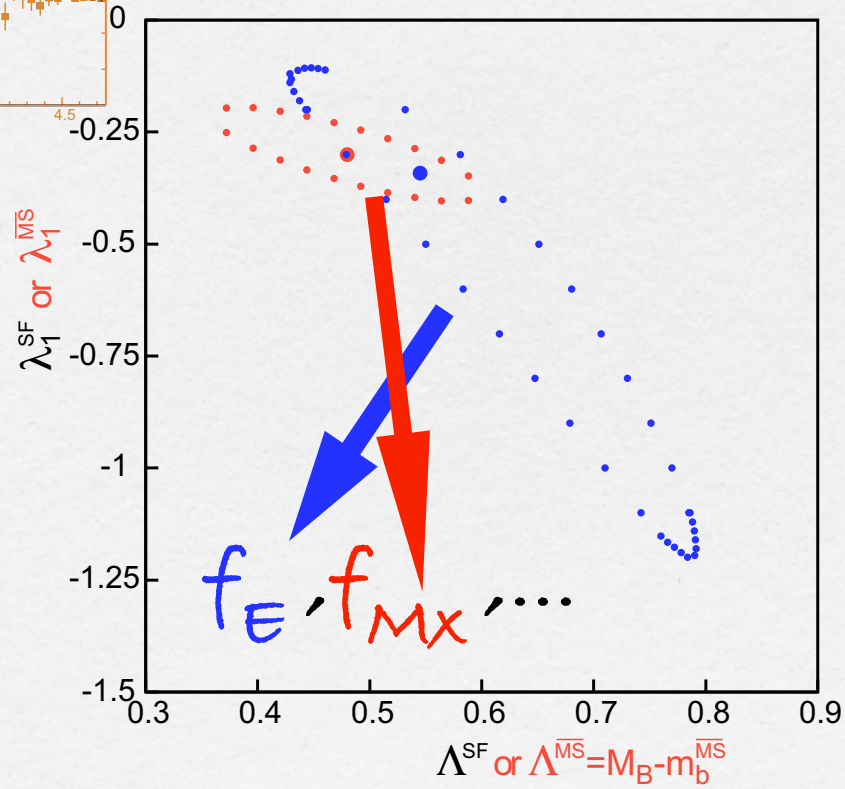

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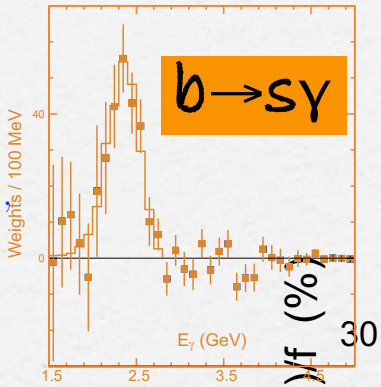
integrals for $b\to u l\nu$, $b\to s\gamma$



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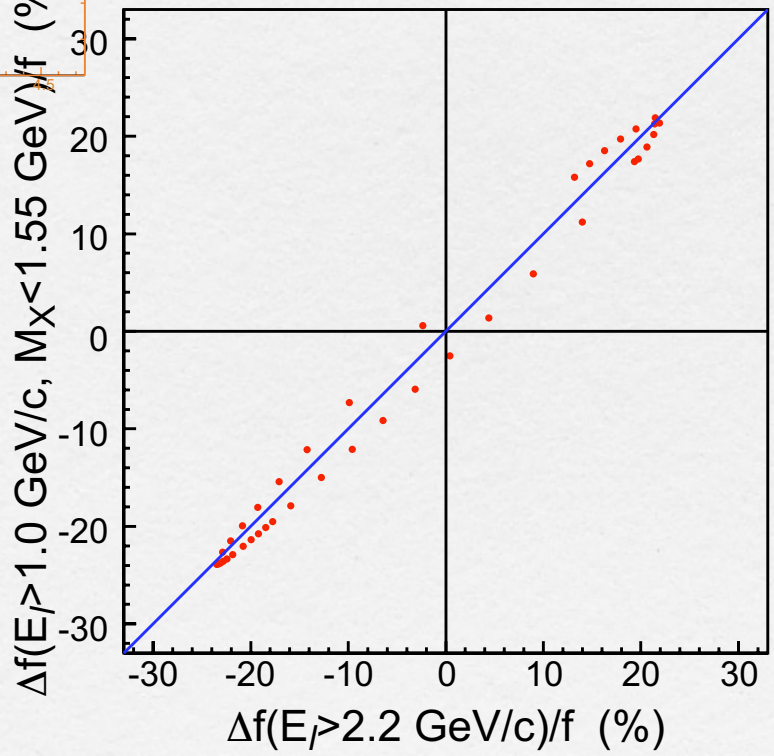
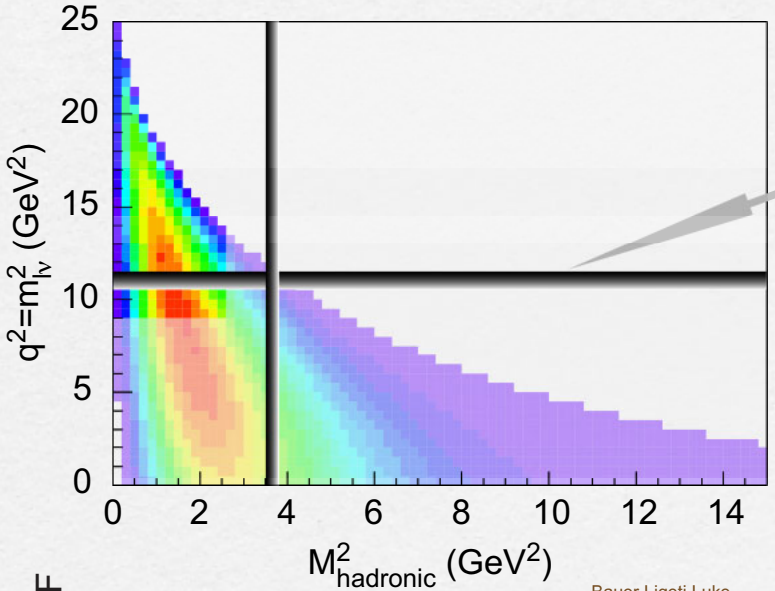
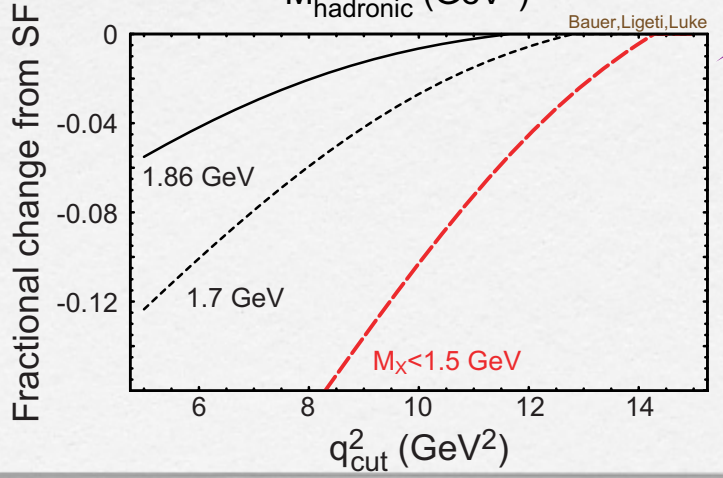


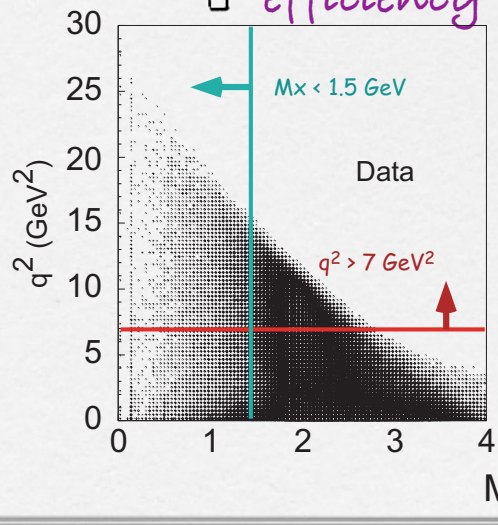
Inclusive: M_X vs q^2

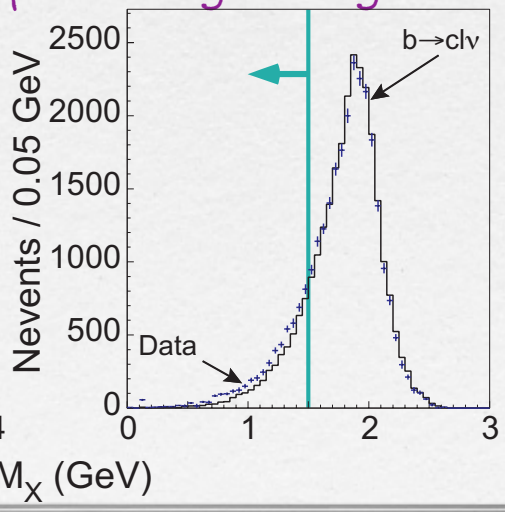


- $q^2 > (M_B - M_D)^2$?
- best bet: combine q^2, M_X cuts
  - more phase space, $1/m^3$ controlled
  - OPE remains valid!
    - minimize shape fcn influence
  - BELLE: `annealing method'
    - efficiency flat in signal region?

$(\Lambda_{QCD}/m_b)^3 \rightarrow (\Lambda_{QCD}/m_c)^3$

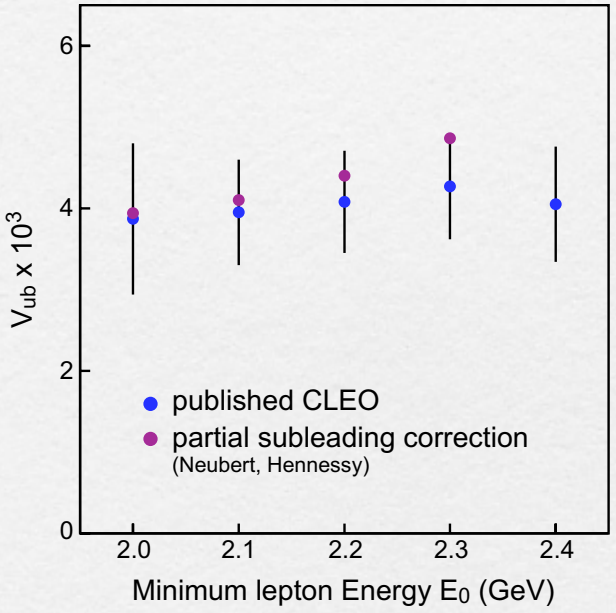


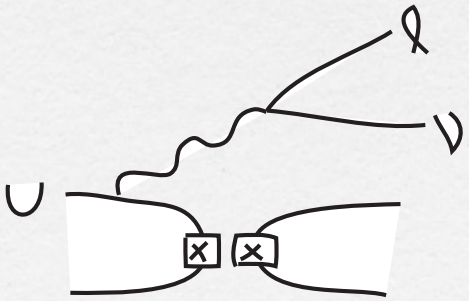




So what's the problem?

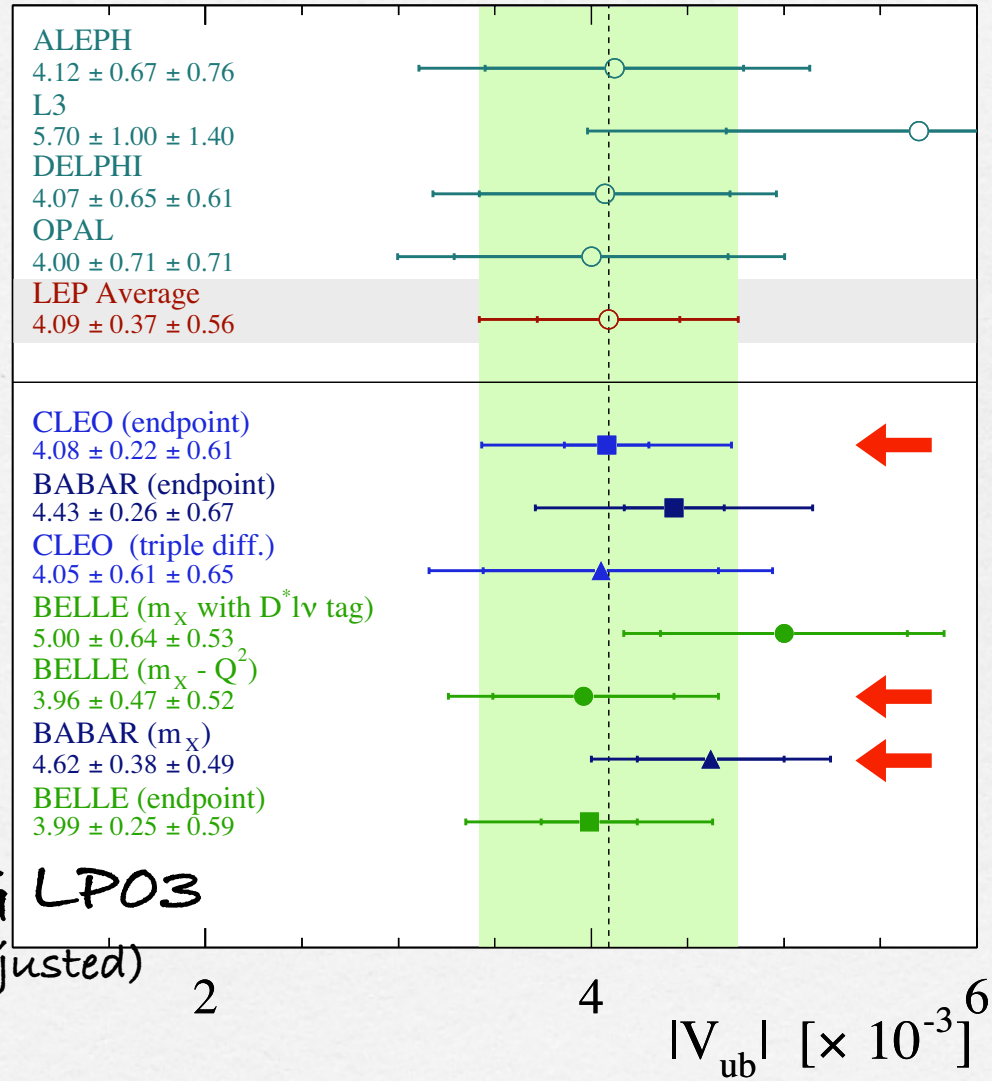
- Subleading corrections to SF
  - $O(\Lambda_{QCD}/M_B) \sim 15\%$
    - F(k+) in ulv
    - ulv, sγ differ
- `Weak annihilation'
  - localized near $q^2 \sim m_b^2$
  - rate α $(16\pi^2) \times$ (factorization viol.)
    - eg.: 10% violation $\Rightarrow \delta\Gamma/\Gamma^{total} \sim 2\text{-}3\%$
      - x10 for endpoint region!
- local duality?





Prototype Combination

1) Pick m'ments w/ phase space sensitive to different effects + well-defined sensitivity
   - correct to common
     - shape function
     - lifetime
     - Γ^{tot}_{thy}



Prototype Combination

2) M_X vs q^2: smallest SF dependence → "central value"

Belle M_X vs q^2: $3.96 \pm 0.17_{stat.} \pm 0.56_{sys.} \pm 0.45_{SF} \pm 0.29_{\Gamma_{thy}} \pm X_{SF_{sub}} \pm X_{WA}$

3) Weak Annihilation: compare "diluted" to "concentrated" effect

BaBar $M_X < 1.55$ GeV: $4.79 \pm 0.29_{stat.} \pm 0.28_{sys.} \pm 0.60_{SF} \pm 0.27_{\Gamma_{thy}} \pm Y_{SF_{sub}} \pm Y_{WA}$

CLEO $E_l > 2.2$ GeV: $4.11 \pm 0.13_{stat.} \pm 0.31_{sys.} \pm 0.51_{SF} \pm 0.23_{\Gamma_{thy}} \pm \ldots$

$$\Delta|V_{ub}| = 0.69 \pm 0.53 \quad \times \quad [(1 - f_{qM})/f_{qM}][f_e f_M/(f_M - f_e)] \approx 0.39$$

$$\Rightarrow \quad \boxed{\sigma_{WA} \approx 0.29}$$

4) subleading SF corr.: compare "more" (BaBar) to "less" (BELLE) SF dependence

$$\Delta|V_{ub}| = 0.83 \pm 0.71 \quad \times \quad (\Delta\Gamma/\Gamma)_{qM}/(\Delta\Gamma/\Gamma)_{M} \sim 0.48$$

$$\Rightarrow \quad \boxed{\sigma_{SF_{sub}} \approx 0.40}$$

Inclusive: proto-combo

17%: ~complete + improvable!

- Include more measurements, more th'y
- Improve $b\to s\gamma$ E_γ spectrum statistics!!!!
- More optimal q^2 range for $M_X<1.5$ GeV
- Other inputs: eg., B^0 vs B^+, D^0 vs D_s limit WA
- th'y: relate mb to Shape Function parameters!

PLEA:

need partial BF's insensitive to detailed $b\to u$ shape

Experimentalists: please quote those rates!!!

$$|V_{ub}| = (3.96 \pm 0.17_{stat} \pm 0.56_{sys} \pm 0.67_{SF_{tot}} \pm 0.29_{\Gamma_{thy}}) \times 10^{-3}$$

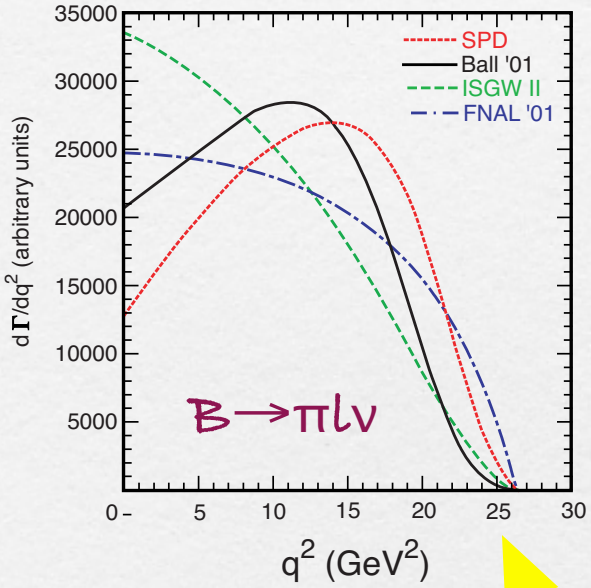
- More q^2 vs M_X^2 measurements!
- higher q^2 optimal? (bkg syst.)

Exclusive: form factors

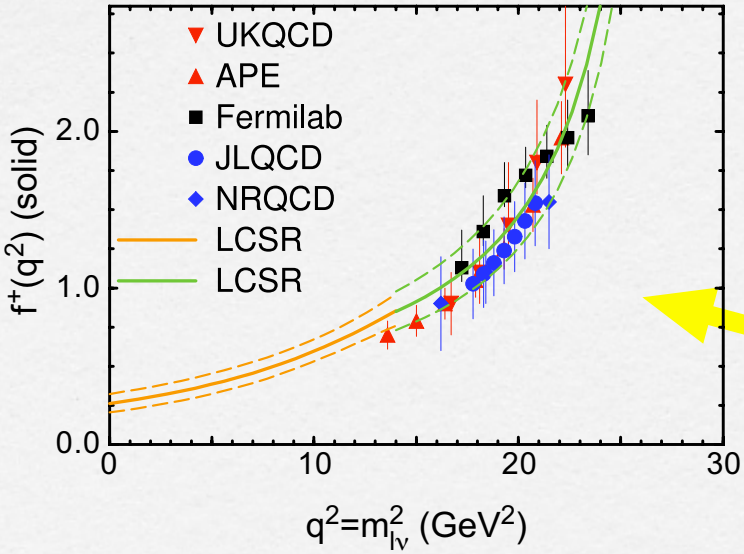
- $|V_{ub}|$ from $B \to \pi/\rho l\nu$
  - Theory = Form factors (FF), eg.

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2\, d\cos\theta_\ell} = |V_{ub}|^2 \frac{G_F^2 p_\pi^3}{32\pi^3} \sin^2\theta_\ell |f^+(q^2)|^2$$

  - shape → rate uncertainty
  - shape+norm → $|V_{ub}|$ uncertainty



Models:
large variation

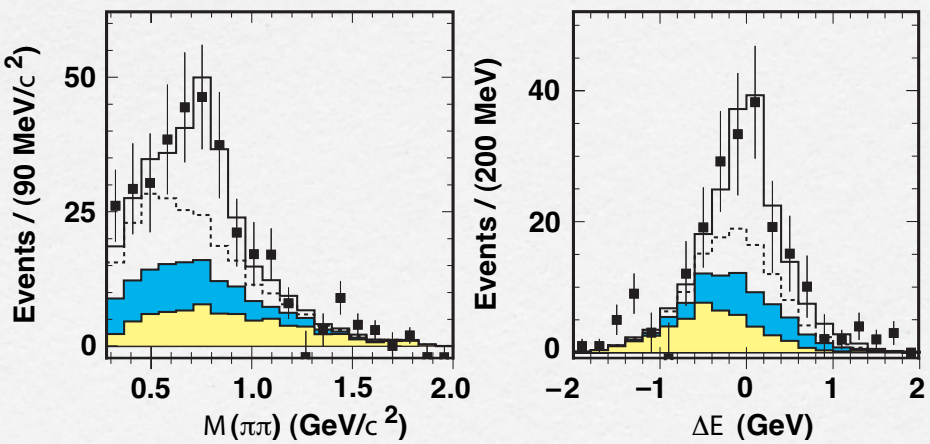


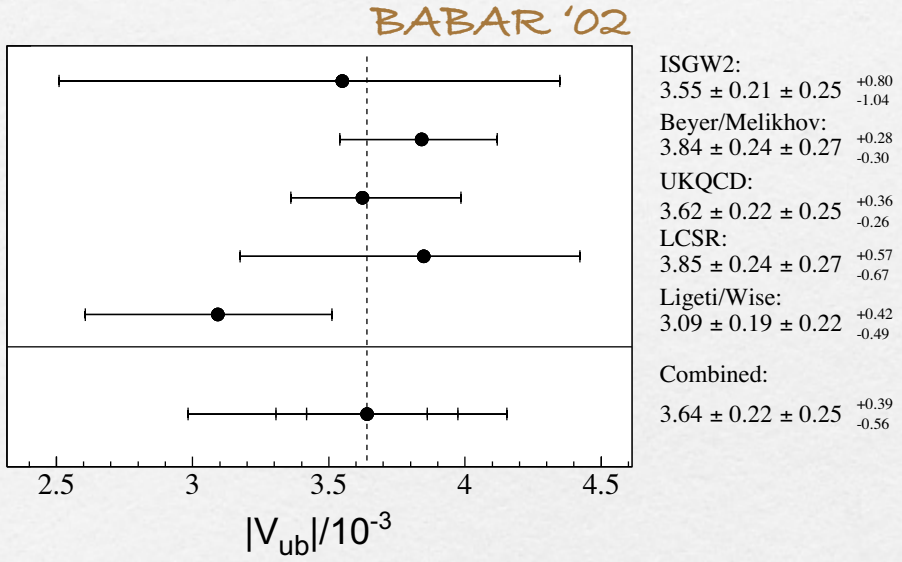
QCD-based:
reasonable agreement
- quenched lattice (high q^2)
- light cone sum rules (low q^2)

"Loose" ν-reconstruction

- Generic technique (to date)
  - Event E_{miss}, p_{miss} → E_ν, p_ν
  - Two approaches:
    - Tight
    - Loose

) event, ν-consistency req's





models + extrapolated QCD FF's ($|V_{ub}|$ error)?

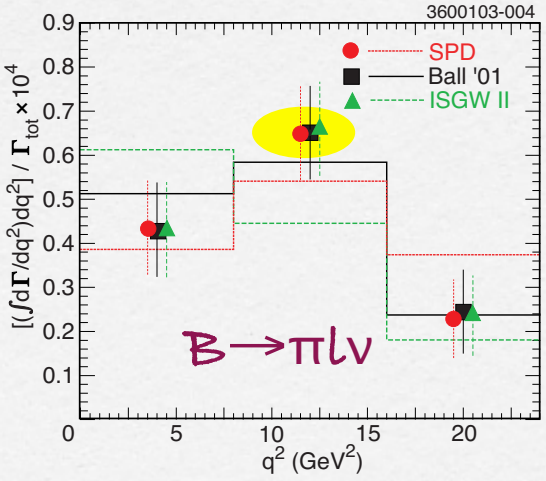
B→plν

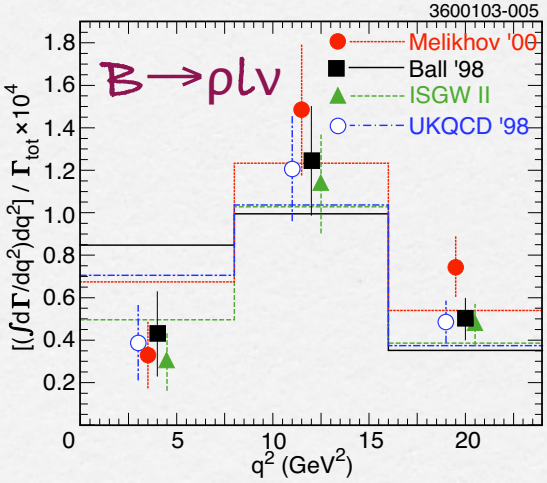
S/B ⇒ primarily sensitive for p_l > 2.3 GeV/c (form factors!)

"Tight" ν-reconstruction

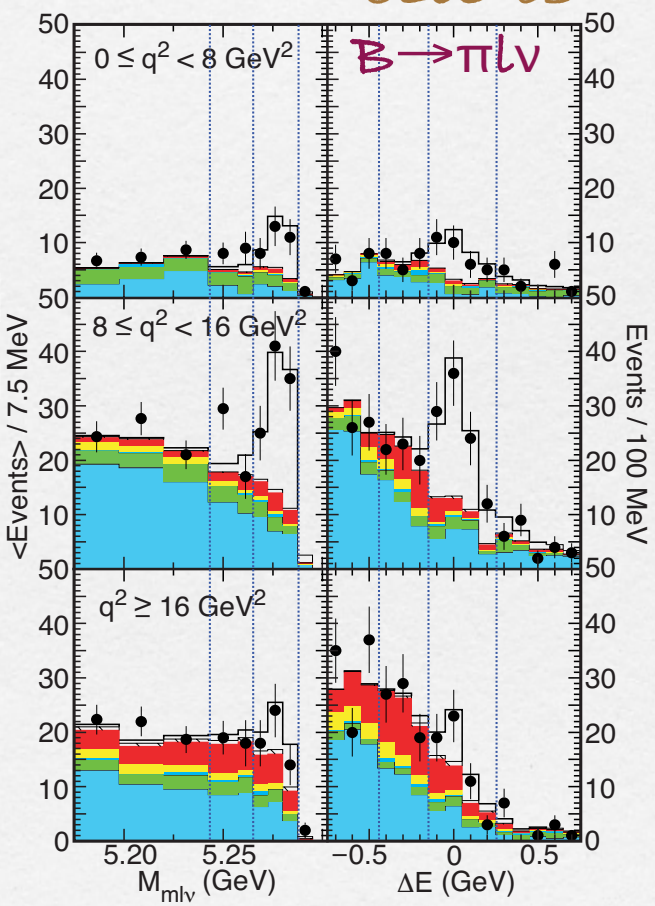
- Better S/N (lower efficiency)
  - larger p_l range
    - >1.0 GeV/c (πlν), 1.5 GeV/c (ρlν)
  - measure $d\Gamma/dq^2$
    - reduce FF shape dependence
    - test FF calc's
    - use QCD calc's, no extrapolation

$B \to \pi/\rho l \nu$

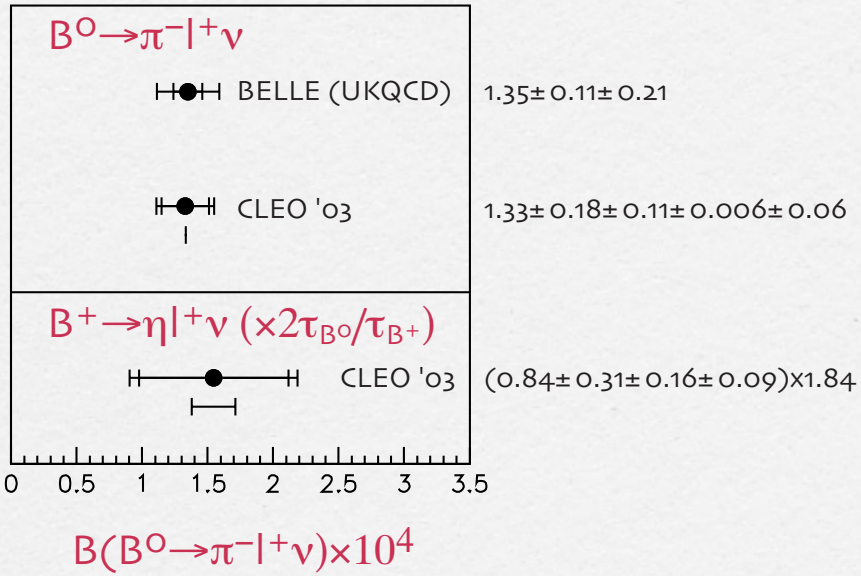


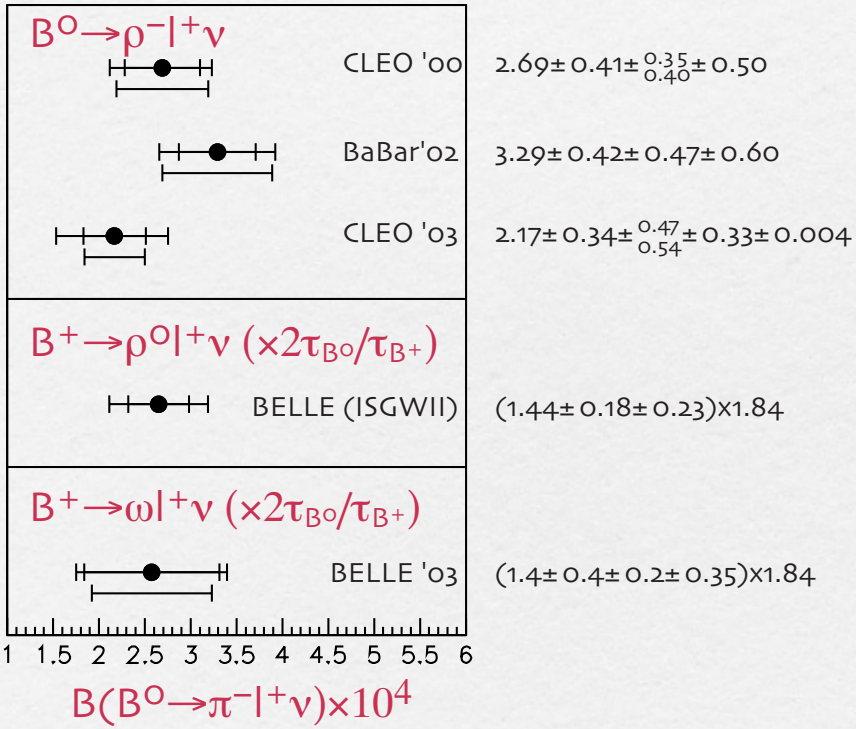


CLEO '03



Branching Fractions



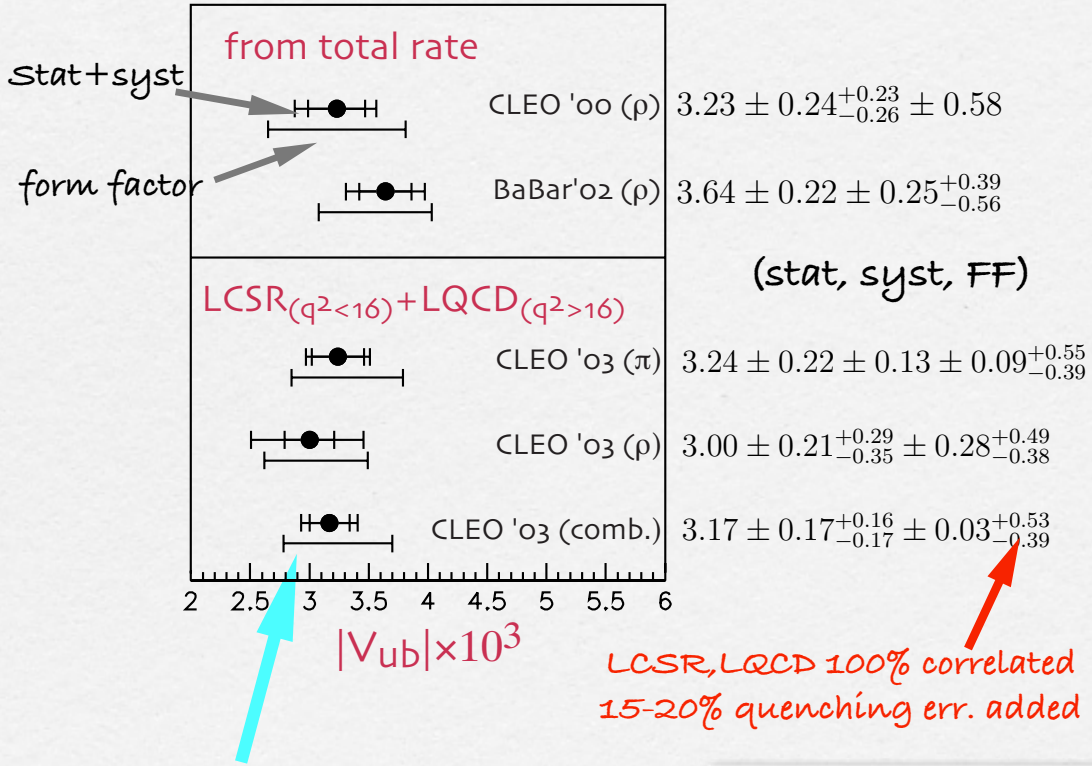


Isospin relations:

$$\Gamma(B^0 \to \pi^- \ell^+ \nu) = 2\Gamma(B^+ \to \pi^0 \ell^+ \nu)$$
$$\Gamma(B^0 \to \rho^- \ell^+ \nu) = 2\Gamma(B^+ \to \rho^0 \ell^+ \nu)$$

- used in CLEO, BABAR π,ρ results
- fits + prelim BELLE modes support

Exclusive: average



$3.23 \pm 0.24^{+0.23}_{-0.26} \pm 0.58$

$3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}$

(stat, syst, FF)

$3.24 \pm 0.22 \pm 0.13 \pm 0.09^{+0.55}_{-0.39}$

$3.00 \pm 0.21^{+0.29}_{-0.35} \pm 0.28^{+0.49}_{-0.38}$

$3.17 \pm 0.17^{+0.16}_{-0.17} \pm 0.03^{+0.53}_{-0.39}$

LCSR,LQCD 100% correlated
15-20% quenching err. added

π dominates
weight ~ 70%

- Correlated syst not evaluated
  - I'll assume 100% corr.
- Modeling versus no modeling?
  - assign 5% deweighting penalty in average
    - admittedly arbitrary

$$|V_{ub}| = (3.27 \pm 0.13 \pm 0.19^{+0.51}_{-0.45}) \times 10^{-3}$$

Average, Prospects

Inclusive errors: all present!

⇒ no longer reason not to combine excl + incl

- one exclusive fudge: additional 10% error added for quenching in average (no expt'l checks)

Prospects: Great!!

- B→πlν looks golden
  - still statistics limited
  - fully recon B tags coming (soon?)
    reduced systematics
  - unquenched lattice + moving B!
    plν harder: unquench⇒unstable

- Much room for improving inclusive procedure outlined here
  - many useful inputs needed from current data!

Average: $|V_{ub}| = (3.48 \pm 0.10 \pm 0.22^{+0.48}_{-0.45}) \times 10^{-3}$