QCD, Factorization, and the Soft-Collinear Effective Theory

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(Beauty 2003)

Outline

- Motiviation, QCD, Expansion Parameters

  Analogy with HQET

- What is the Soft-Collinear Effective Theory (SCET)?

  Degrees of freedom, Physical picture, Symmetries

- B-Physics Applications: ($m_B \simeq 5.3\,\mathrm{GeV}$)

  $B \to D\pi$, Color-Suppressed Decays ($\bar{B}^0 \to D^0\pi^0, \ldots$)

  Heavy-to-Light Form factors

  Status of $B \to MM$ decays ($B \to \pi\pi, \ldots$)

- Outlook and Issues

  Summary from Beauty-SCET Workshop

Motivation

b-Hadrons:

- Laboratory for EW, new physics, & QCD
- The lightest B's decay weakly to many channnels

$$B \to D^{(*)} e\nu,\ B \to D_{1,2}^{(*)} e\nu,\ B \to \pi e\nu,\ B \to \rho e\nu,$$
$$B \to K^*\gamma,\ B \to K e^+ e^-,\ B \to \rho\gamma,$$
$$B \to \tau\nu,\ B \to \gamma e\nu,\ B \to e^+ e^- e\nu,$$
$$B \to D\pi,\ B \to \pi\pi,\ B \to K\pi,\ B \to J/\Psi K_S,$$
$$B \to X_u e\nu,\ B \to X_s\gamma,\ B \to X_s \nu\bar{\nu},$$
$$\ldots$$

(Repeat for B_s, Λ_b, $\ldots$)

▷ Need to understand (elliminate) hadronic uncertainties from QCD

Scales

quarks	mass
u	$\sim 4\,\mathrm{Mev}$
d	$\sim 7\,\mathrm{MeV}$
s	$\sim 120\,\mathrm{MeV}$
c	$\sim 1.4\,\mathrm{GeV}$
b	$\sim 4.5\,\mathrm{GeV}$
t	$174\,\mathrm{GeV}$

u, d, s } light

$\Longleftarrow \Lambda$

c, b, t } heavy

QCD

$\alpha_s(\mu)$, μ resolution

$\alpha_s(\Lambda)$ non-perturbative
$\rightarrow$ long distance

$\alpha_s(m_b)$ perturbative
$\rightarrow$ short distance

In full QCD usually we can not predict amplitudes with small uncertainties in a model independent way

Need Expansion Parameters

(If we use a model then we can not even estimate the uncertainties reliably)

Use Effective Field Theories

Use Effective Field Theories: Separate physics at different momentum scales

Expansion Parameters

(1) Electroweak Hamiltonian

$m_b/m_W \ll 1$

(2) Heavy Quark Effective Theory (HQET)

$\Lambda/m_b \ll 1$

(3) SU(3), Chiral Perturbation Theory

$m_{u,d,s}/\Lambda \ll 1$

(4) Soft-Collinear Effective Theory (SCET)

$\Lambda/Q \ll 1, \quad Q = \{m_b, E_H\}$

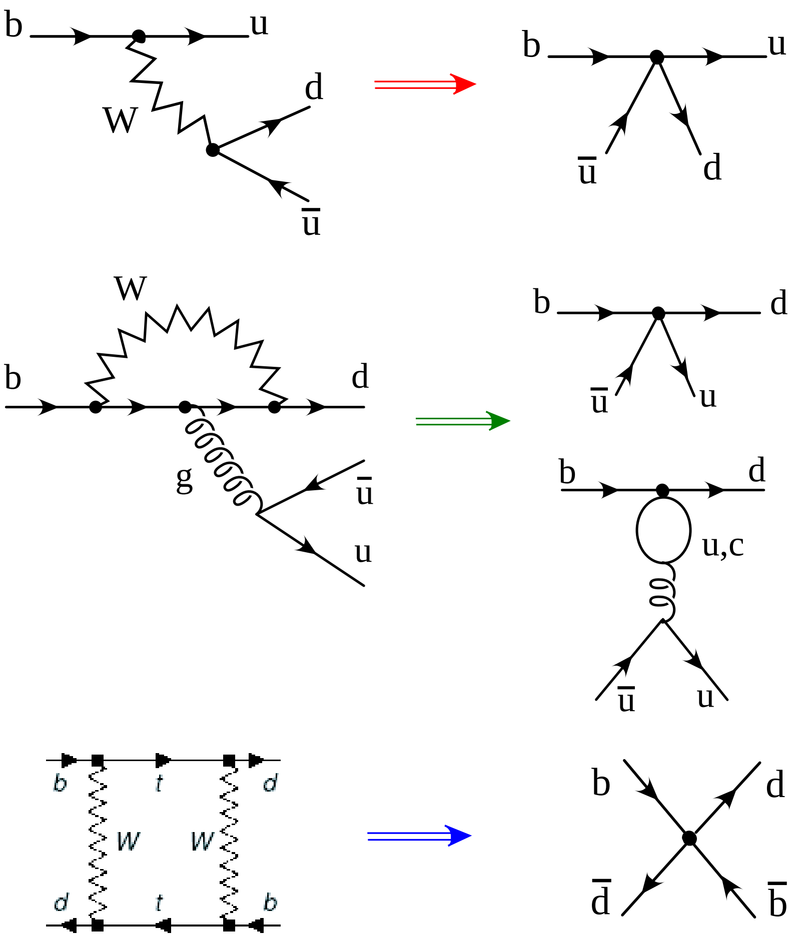
Depending on the observable one or more of these may be necessary

(5) Lattice QCD

$a/r \ll 1, \; r^3/V \ll 1$

Electroweak Hamiltonian

1) Integrating out the W, t ($m_W, m_t \gg m_b$):



$$H_W = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$$

$$O_1 = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}$$

$$O_2 = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}$$

$$O_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$O_{4,5,6} = \ldots$$

$$O_{7\gamma,8G} = \ldots$$

$$O^{ew}_{7,\ldots,10} = \ldots$$

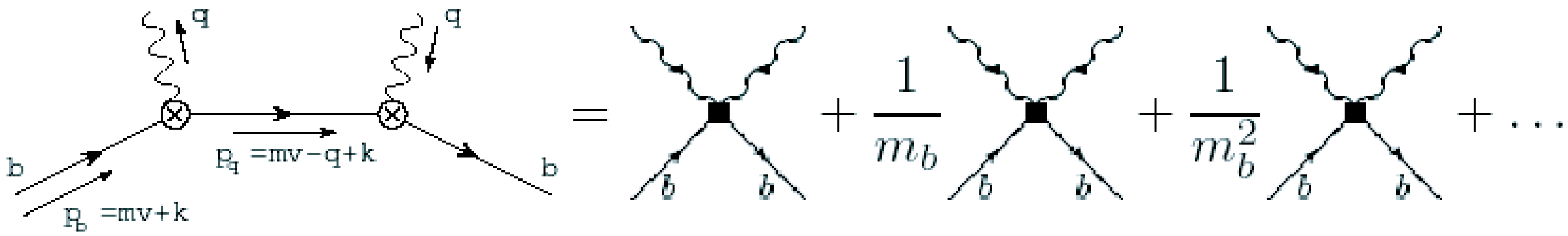
$$O_{\Delta B=2} = (\bar{d}b)_{V-A}(\bar{d}b)_{V-A}$$

$$\ldots$$

Operators come with different CKM elements, $\lambda^1 = V_{ub}V^*_{ud}, \ldots$

HQET: $B \to X_c \ell \bar{\nu}$

Inclusive Decay: OPE in Λ/m_b



- $m_b \to \infty$ is free quark decay, $\alpha_s(m_b)$ corrections computable
- No Λ/m_b corrections $\to$ HQET gives 0 at this order
- At Λ^2/m_b^2 have dependence on λ_1, λ_2 defined in HQET

$$\lambda_1 = -\frac{1}{2}\langle B_v|\bar{h}_v D_\perp^2 h_v|B_v\rangle \,,$$
$$\lambda_2 = \dots$$

HQET is simpler than QCD $\to$ Spin-Flavor Symmetry

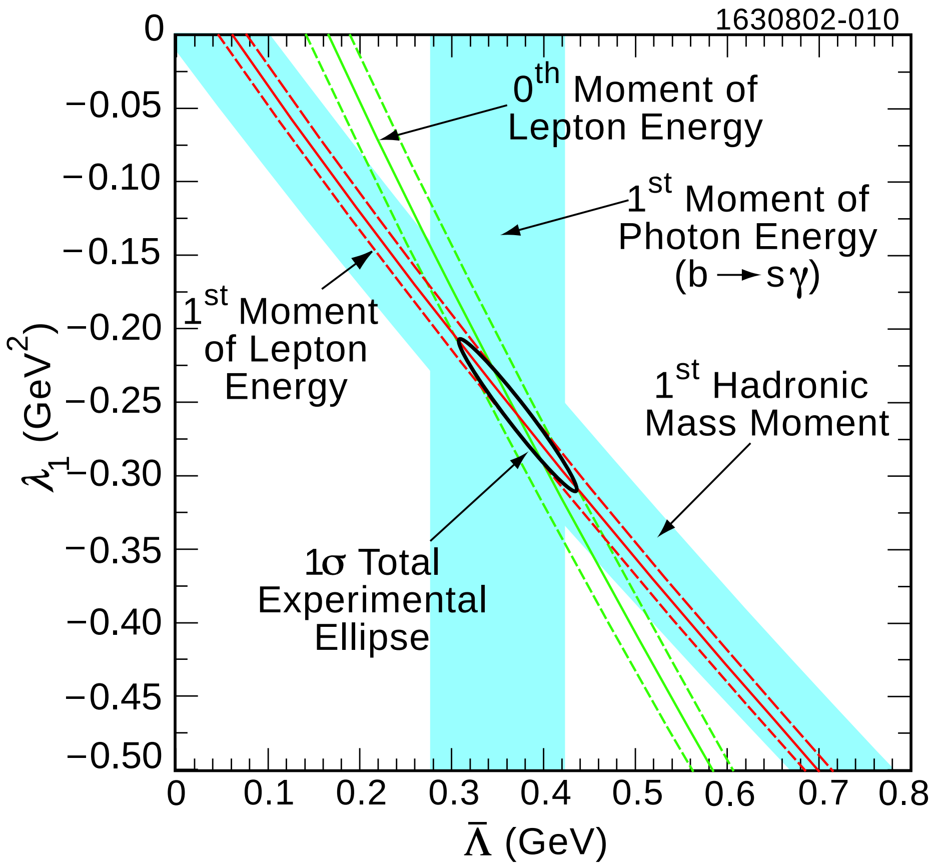
Uncertainties suppressed by Λ/m_b

HQET: $B \to X_c \ell \bar{\nu}$

Inclusive Decay: OPE in Λ/m_b

- Fit moments to simultaneously extract $|V_{cb}|$, m_b ($\bar{\Lambda}$), λ_1, λ_2

Example: CLEO



$$|V_{cb}| = (40.8 \pm 0.6 \pm 0.9) \times 10^{-3}$$

from S.Stone at EPS

Soft-Collinear Effective Theory

Many processes have energetic hadrons, $Q \gg \Lambda$, where HQET does not apply

Soft-Collinear Effective Theory

C. Bauer, S. Fleming, M. Luke — hep-ph/0005275 (PRD)
C. Bauer, S. Fleming, D. Pirjol, I.S. — hep-ph/0011336 (PRD)
C. Bauer, I.S. — hep-ph/0107001 (PLB)
C. Bauer, D. Pirjol, I.S. — hep-ph/0109045 (PRD)

Builds on earlier work:

Hard Exclusive: Brodsky, Lepage, . . .

Jet physics: Collins, Soper, Sterman, Korchemsky, . . .

B-physics Factorization: Dugan, Grinstein, Beneke, Buchalla, Neubert, Sachrajda, . . .

Soft-Collinear Effective Theory

Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Offshell modes with $p^2 \gg Q^2\lambda^2$ are integrated out (in coefficients)

Soft-Collinear Effective Theory

Introduce fields for infrared degrees of freedom (in operators)

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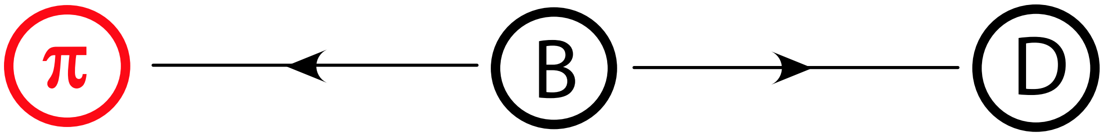
eg.



Pion has: $p_\pi^\mu \; = \; (2.3\,\mathrm{GeV})n^\mu = Q\, n^\mu \qquad n^2 = \bar{n}^2 = 0,\ (\bar{n}\cdot p_\pi = 2Q)$

pion in rest frame has constituent momenta:

$$(p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda)$$

boosting gives collinear constituents:

$$(p^+, p^-, p^\perp) \sim \Big(\frac{\Lambda^2}{Q}, Q, \Lambda\Big) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda \ll 1$$

Soft-Collinear Effective Theory

Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+,-,\perp)$	p^2	fields
collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda,\lambda,\lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
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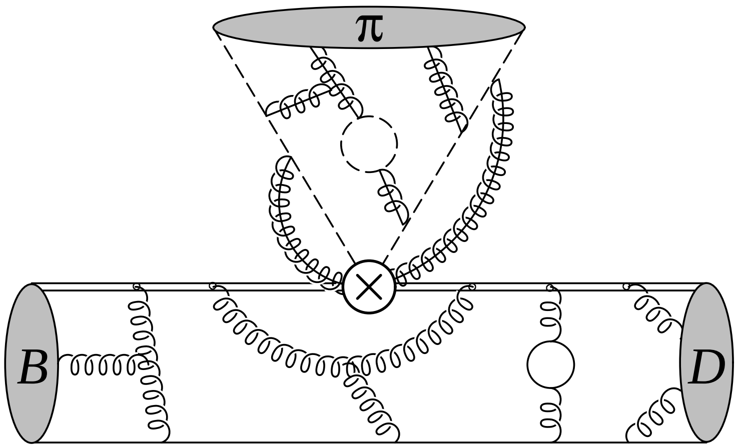

B, D are soft, π collinear

$$\mathcal{L} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$$

Factorization if $\mathcal{O} = O_1 \times O_2$

Soft-Collinear Effective Theory

Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+,-,\perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Typically either:

SCET$_{\rm I}$ $\quad \lambda = \sqrt{\Lambda/Q}$ $\longrightarrow$ usoft $p^\mu \sim \Lambda$; collinear $p_c^2 \sim Q\Lambda$, jets

SCET$_{\rm II}$ $\quad \lambda = \Lambda/Q$ $\longrightarrow$ soft $p^\mu \sim \Lambda$; collinear $p_c^2 \sim \Lambda^2$, exclusive

Soft-Collinear Effective Theory

Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Symmetries

1) Gauge Symmetry, Collinear, Soft, Usoft
2) Helicity, Spinor Reduction, $\not{n}\xi_n = 0$
3) Reparameterization Invariance, n, $\bar{n}$
4) C,P,T in different sectors

SCET

- gives a systematic expansion in $\lambda \sim \Lambda_{\rm QCD}/Q$
- model independent description of power corrections

  can estimate uncertainties

- make symmetries explicit, understand factorization in a universal way

Determine quantities that are short and long distance,
calculate short distance coefficients

Proof of Factorization means Known to be Model Independent once hadronic parameters are determined

- $\mathrm{SCET_I}$ has hard coefficients $C(\bar{\mathcal{P}},\mu)$ with $\mu^2 \sim Q^2$, Wilson lines W, Y
- $\mathrm{SCET_{II}}$ has jet coefficients J with $\mu^2 \sim Q\Lambda$, Wilson lines W, S

Hadronic Parameters

Define universal hadronic parameters, exploit symmetries

Process	Degrees of Freedom (p^2)	Non-Pert. functions
$\bar{B}^0 \to D^+\pi^-, \ldots$	c (Λ^2), s (Λ^2)	$\xi(w)$, ϕ_π
$\bar{B}^0 \to D^0\pi^0, \ldots$	c (Λ^2), s (Λ^2), c ($Q\Lambda$)	$S(k_j^+)$, ϕ_π
$B \to X_s^{endpt}\gamma$,	c ($Q\Lambda$), us (Λ^2)	$f(k^+)$
$B \to X_u^{endpt}\ell\nu$		
$B \to \pi\ell\nu, \ldots$	c ($Q\Lambda$), s (Λ^2), c (Λ^2)	$\phi_B(k^+)$, $\phi_\pi(x)$, $\zeta_\pi(E)$
$B \to \gamma\ell\nu$	c ($Q\Lambda$), us (Λ^2)	ϕ_B
$B \to \pi\pi$	c (Λ^2), s (Λ^2), c ($Q\Lambda$)	ϕ_B, ϕ_π, $\zeta_\pi(E)$
$B \to K^*\gamma$	c ($Q\Lambda$), s (Λ^2), c (Λ^2)	ϕ_B, ϕ_K, $\zeta_{K^*}^\perp(E)$
$e^-p \to e^-X$	c (Λ^2)	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$
$e^-\gamma \to e^-\pi^0$	c (Λ^2), s (Λ^2)	ϕ_π
$\gamma^*M \to M'$	c (Λ^2), s (Λ^2)	ϕ_M, $\phi_{M'}$

Hadronic Parameters

SCET Authors (in no particular order):

S.Mantry, C.Bauer, D.Pirjol, I.S., S.Fleming, M.Luke, I.Rothstein, M.Beneke, T.Feldmann, M.Diehl, A.Chapovsky, Descotes-Genon, J.Chay, C.Kim, G.Buchalla, C.Sachrajda, E.Lunghi, D.Wyler, S.Bosch, R.Hill, B.Lange, M.Neubert, T.Becher, M.Wise, A.Manohar, T.Mehen, A.Leibovich, Z.Ligeti, . . .

Hadronic Parameters

Example: $\bar{B}^0 \to D^+\pi^-$, $B^- \to D^0\pi^-$

$$\langle D\pi|\bar{c}b\bar{u}d|B\rangle = N\,\xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)$$

where

$$\langle \pi_n|\bar{\xi}^{(0)}_{n,p'}\,W^{(0)}C_{\mathbf{O}}(\bar{\mathcal{P}}_+)W^{(0)\dagger}\xi^{(0)}_{n,p}|0\rangle = \frac{i}{2}f_\pi E_\pi\int dx\, C[2E_\pi(2x-1)]\phi_\pi(x)$$

$$\langle D_{v'}|\bar{h}_{v'}\Gamma_h h_v|B_v\rangle = \xi(v\cdot v')$$

LO = λ^5 graphs

$Q = m_b, m_c, E_\pi \gg \Lambda$, corrections will be $\Lambda/m_c \sim 30\%$

Example 2: $B \to X_s\gamma$

shape function $\quad f(l^+) = \langle B|\bar{h}_v\delta(in\cdot D - l^+)h_v|B\rangle$

$B \to D^{(*)} X$ phenomenology

Type	Decay	Br(10^{-3})	Decay	Br(10^{-3})
I	$\bar{B}^0 \to D^+ \pi^-$	2.68 ± 0.29 [a]	$\bar{B}^0 \to D^{*+} \pi^-$	2.76 ± 0.21
III	$B^- \to D^0 \pi^-$	4.97 ± 0.38 [a]	$B^- \to D^{*0} \pi^-$	4.6 ± 0.4
II	$\bar{B}^0 \to D^0 \pi^0$	0.292 ± 0.045 [b]	$\bar{B}^0 \to D^{*0} \pi^0$ [b]	0.25 ± 0.07
I	$\bar{B}^0 \to D^+ \rho^-$	7.8 ± 1.4	$\bar{B}^0 \to D^{*+} \rho^-$	6.8 ± 1.0 [c]
III	$B^- \to D^0 \rho^-$	13.4 ± 1.8	$B^- \to D^{*0} \rho^-$	9.8 ± 1.8 [c]
II	$\bar{B}^0 \to D^0 \rho^0$	0.29 ± 0.11 [d]	$\bar{B}^0 \to D^{*0} \rho^0$	< 0.56

PDG or a,b,c,d=CLEO, b=BELLE

New BaBar numbers (hep-ex/0310028 , yesterday):

$\bar{B}^0 \to D^0 \pi^0$ $\quad 0.29 \pm 0.02 \pm 0.03$

$\bar{B}^0 \to D^{*0} \pi^0$ $\quad 0.29 \pm 0.04 \pm 0.05$

$B \to D^{(*)} X$ phenomenology

Type	Decay	Br(10^{-3})	Decay	Br(10^{-3})
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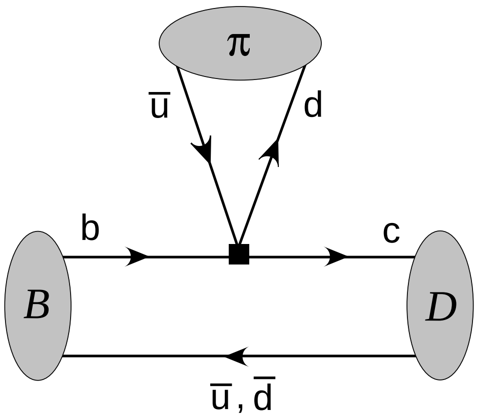
- $\bar{B}^0 \to D^{(*)+} P^-$ decays agree within errors
- $\bar{B}^0 \to D^{(*)0} \pi^0$, $\bar{B}^0 \to D^{(*)0} \rho^0$, small as expected (0 at LO)
- $\sim$ 20-30% power corrections for $A(B^- \to D^0 P^-)/A(\bar{B}^0 \to D^+ P^-)$
  Nonzero strong phase, $\delta \sim 30^\circ$

$B \to D\pi$

$$O^0 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$
$$O^8 = (\bar{c}T^A b)_{V-A}(\bar{d}T^A u)_{V-A}$$

"Tree"



$$\bar{B}^0 \to D^+\pi^-$$
$$B^- \to D^0\pi^-$$

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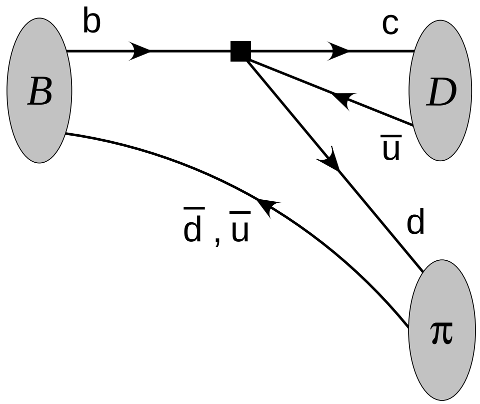
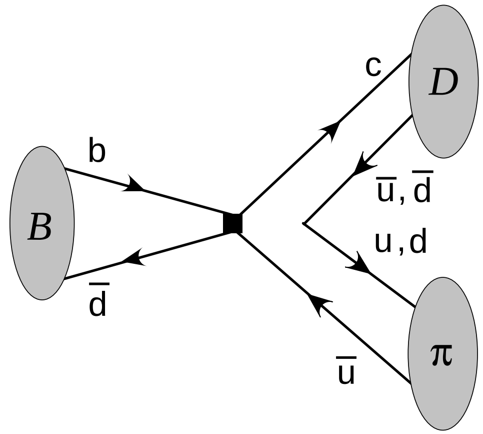


$$B^- \to D^0\pi^-$$
$$\bar{B}^0 \to D^0\pi^0$$

"Exchange"



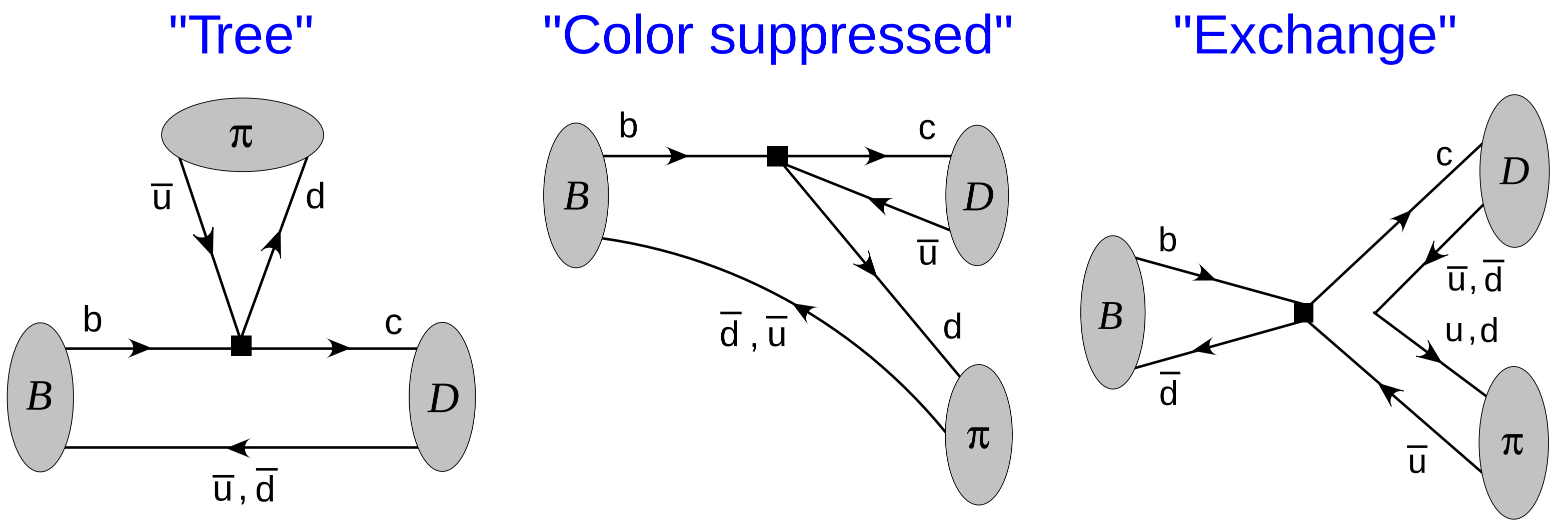
$$\bar{B}^0 \to D^+\pi^-$$
$$\bar{B}^0 \to D^0\pi^0$$

Large N_c - not very predictive:

$(N_c)^0$ $\quad$ $1/N_c$ $\quad$ $1/N_c$

$B \to D\pi$

$$O^0 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$
$$O^8 = (\bar{c}T^A b)_{V-A}(\bar{d}T^A u)_{V-A}$$



"Tree"	"Color suppressed"	"Exchange"
$\bar{B}^0 \to D^+\pi^-$	$B^- \to D^0\pi^-$	$\bar{B}^0 \to D^+\pi^-$
$B^- \to D^0\pi^-$	$\bar{B}^0 \to D^0\pi^0$	$\bar{B}^0 \to D^0\pi^0$

Naive Factorization - too small:

$$A(\bar{B}^0 \to D^0\pi^0) \sim a_2\langle\pi^0|(\bar{d}b)|\bar{B}^0\rangle\langle D^0|(\bar{c}u)|0\rangle$$

$B \to D\pi$

$$O^0 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$
$$O^8 = (\bar{c}T^A b)_{V-A}(\bar{d}T^A u)_{V-A}$$

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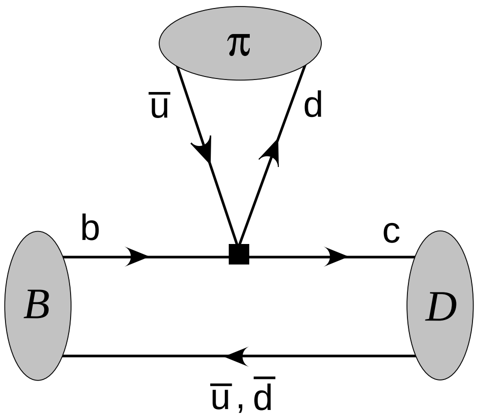

$\bar{B}^0 \to D^+\pi^-$

$B^- \to D^0\pi^-$

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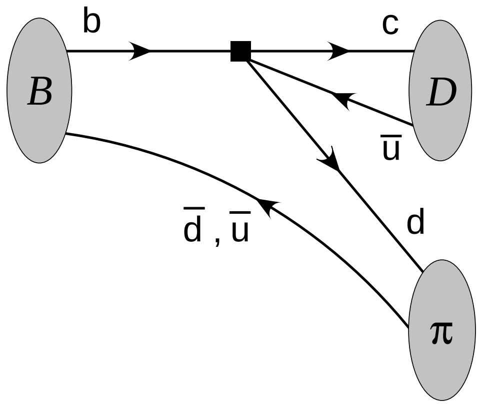
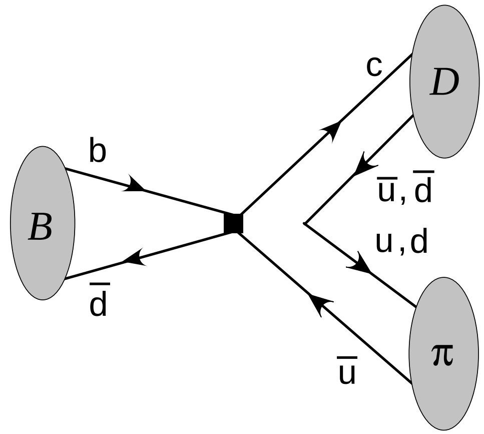


$B^- \to D^0\pi^-$

$\bar{B}^0 \to D^0\pi^0$

"Exchange"



$\bar{B}^0 \to D^+\pi^-$

$\bar{B}^0 \to D^0\pi^0$

QCDF - $D^0\pi^0$ is non-factorizable channel — BBNS

pQCD - predicted with expansion in m_c/m_b — Keum et.al.

Factorization for Color-Suppressed Decays

$\bar{B}^0 \to D^{(*)0} M^0$

Mantry, Pirjol, I.S.

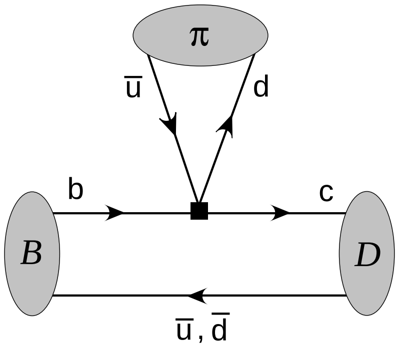
- SCET factorization mechanism for color suppressed channels

still predictive

Factorization for Color-Suppressed Decays

Mantry, Pirjol, I.S.

"Tree"



$\bar{B}^0 \to D^+\pi^-$

$B^- \to D^0\pi^-$

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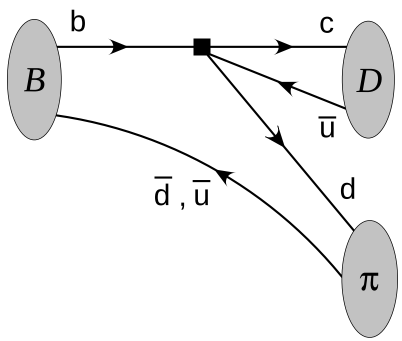
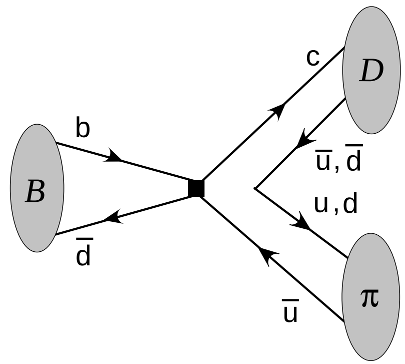


$B^- \to D^0\pi^-$

$\bar{B}^0 \to D^0\pi^0$

"Exchange"



$\bar{B}^0 \to D^+\pi^-$

$\bar{B}^0 \to D^0\pi^0$

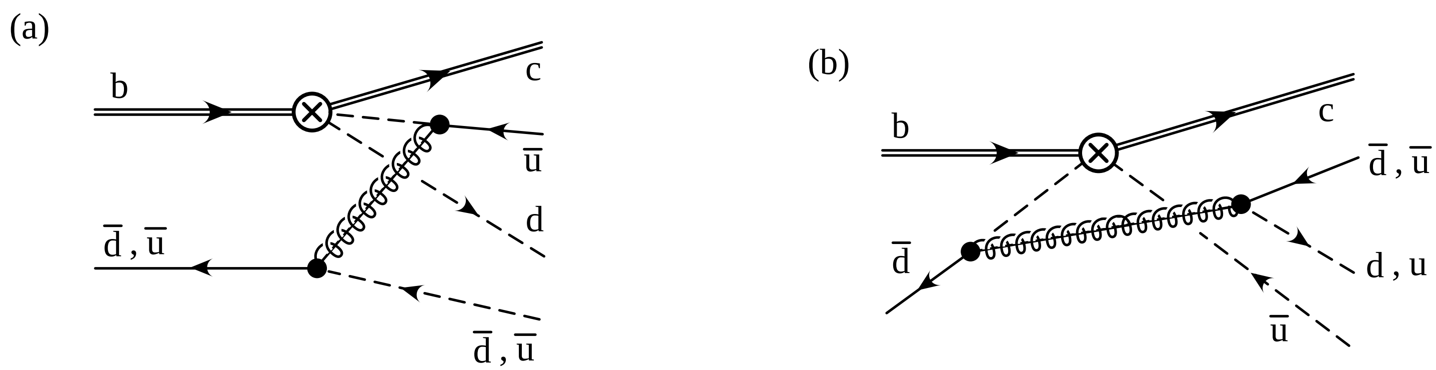
$$A(\bar{B}^0 \to D^+\pi^-) = \frac{1}{\sqrt{3}}A_{3/2} + \sqrt{\frac{2}{3}}A_{1/2} = T + E$$

$$A(B^- \to D^0\pi^-) = \sqrt{3}A_{3/2} = T + C$$

$$A(\bar{B}^0 \to D^0\pi^0) = \sqrt{\frac{2}{3}}A_{3/2} - \frac{1}{\sqrt{3}}A_{1/2} = \frac{1}{\sqrt{2}}(C - E) \equiv A_0$$

Take $E_\pi \gg \Lambda_{\rm QCD}$

Mediated by a single class of $\mathrm{SCET_I}$ operators $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$



When matched onto $\mathrm{SCET_{II}}$ we find a factorization formula ($M=\pi,\rho$):

$$A_{00}^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}_{L\mp R}(z)\, J^{(i)}(z,x,k_1^+,k_2^+)\, S^{(i)}(k_1^+,k_2^+)\, \phi_M(x)$$

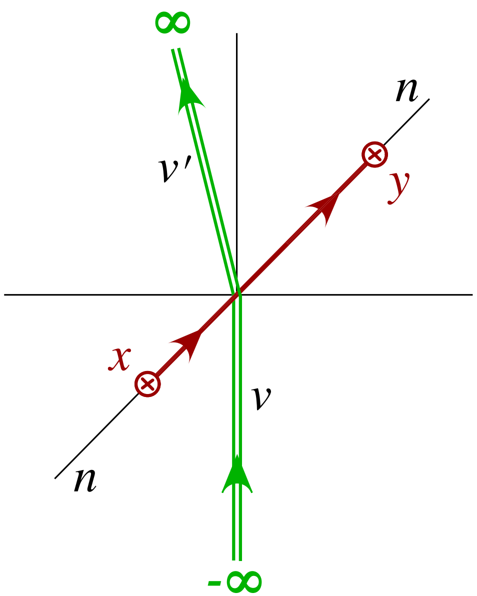
two new non-perturbative soft functions $\qquad (i=0,8)$

$$O^{(0,8)} = \Big[(\bar h_{v'}^{(c)} S)\Gamma^h\{1,T^a\}\,(S^\dagger h_v^{(b)})\,(\bar d\, S)_{k_1^+}\Gamma_s\{1,T^a\}(S^\dagger u)_{k_2^+} \Big]$$

$\langle D^{(*)0}|O^{(0,8)}|\bar B^0\rangle \to S^{(0,8)}(k_1^+,k_2^+)$ same for D and D^*

Results and Predictions

- Find both C and E suppressed by Λ/Q relative to T
- $S(k_i^+)$ is complex, gives non-perturbative strong phase which is independent of M and choice of D vs. D^*



Predict

equal strong phases $\delta^D = \delta^{D^*}$

equal amplitudes $A_0^D = A_0^{D*}$

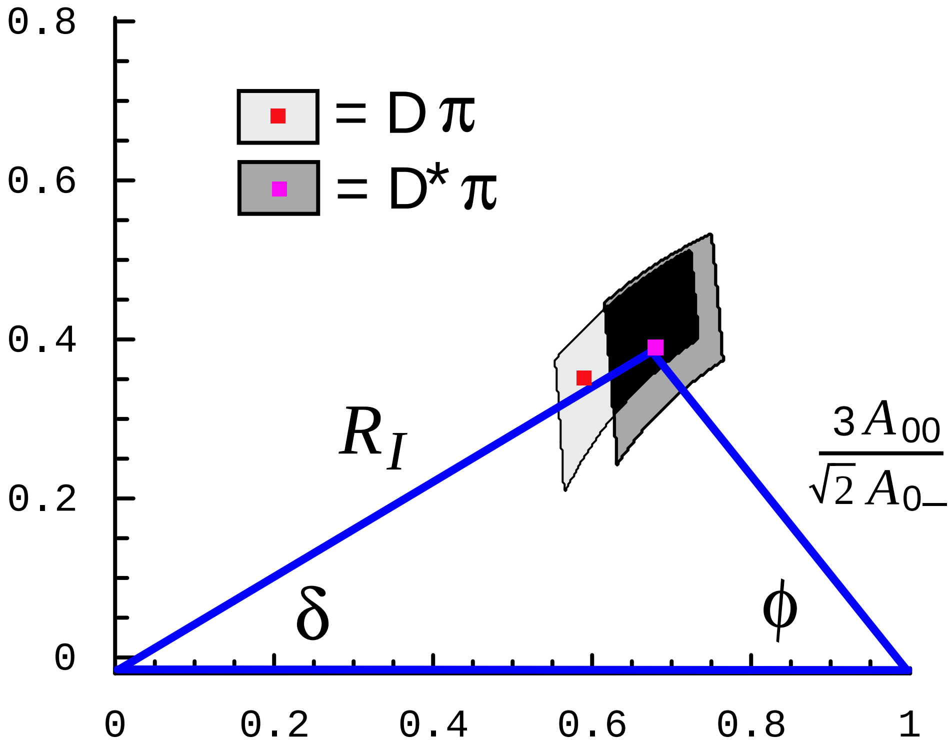
corrections to this are $\alpha_s(m_b)$, Λ/Q

Expt (pdg average):

$$Br(\bar{B}^0 \to D^0\pi^0) = (0.29 \pm 0.05) \times 10^{-3}\ , \qquad \delta^{D\pi} = 30^\circ{}^{+8^\circ}_{-14^\circ}$$

$$Br(\bar{B}^0 \to D^{*0}\pi^0) = (0.25 \pm 0.07) \times 10^{-3}\ , \qquad \delta^{D^*\pi} = 30^\circ \pm 6^\circ$$

Test and Predictions



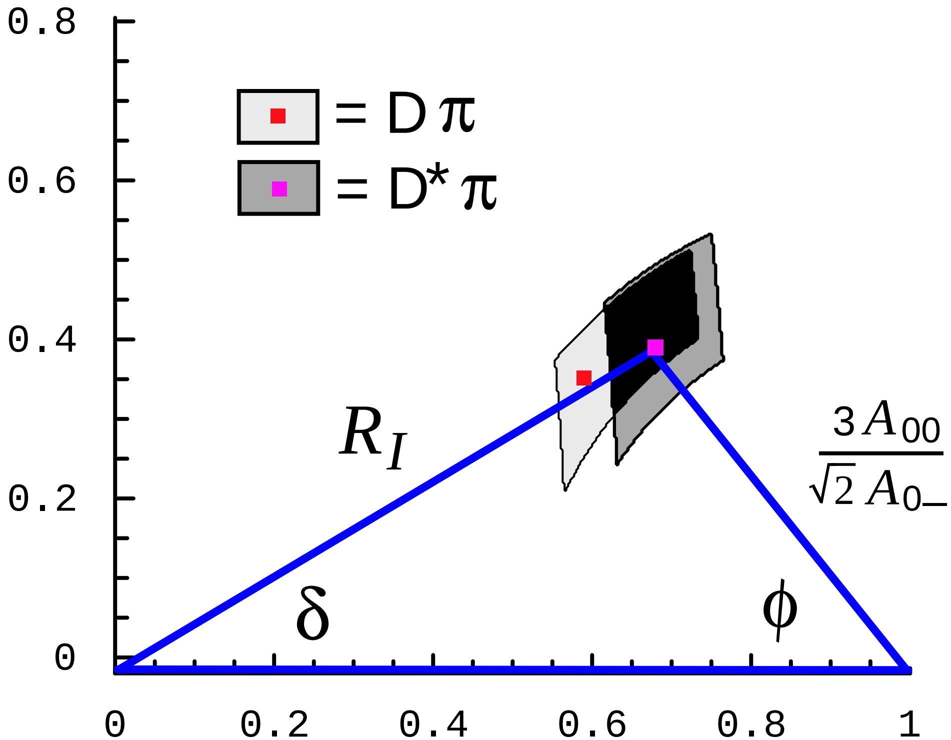
$$R_I = \frac{A_{1/2}}{\sqrt{2}A_{3/2}}$$

$$\delta = \arg(A_{1/2}A^*_{3/2})$$

Also predict (not post-dict):

$$R_0^\rho = \frac{A(\bar{B}^0 \to D^{*0}\rho^0)}{A(\bar{B}^0 \to D^0\rho^0)} = 1\,,$$

Test and Predictions



$$R_I = \frac{A_{1/2}}{\sqrt{2}A_{3/2}}$$

$$\delta = \arg(A_{1/2}A^*_{3/2})$$

Also predict (not post-dict):

$$R_0^{K^-} = \frac{A(\bar{B}^0 \to D_s^* K^-)}{A(\bar{B}^0 \to D_s K^-)} = 1\,, \qquad R_0^{K_\parallel^{*-}} = \frac{A(\bar{B}^0 \to D_s^* K_\parallel^{*-})}{A(\bar{B}^0 \to D_s K_\parallel^{*-})} = 1\,,$$

$$R_0^{K^0} = \frac{A(\bar{B}^0 \to D^{0*} \bar{K}^0)}{A(\bar{B}^0 \to D^0 \bar{K}^0)} = 1\,, \qquad R_0^{K_\parallel^{*0}} = \frac{A(\bar{B}^0 \to D^{*0} \bar{K}_\parallel^{*0})}{A(\bar{B}^0 \to D^0 \bar{K}_\parallel^{*0})} = 1$$

More Predictions

More predictions can be made if we expand J in $\alpha_s(\mu_0^2 = Q\Lambda)$

At tree level $T^{(i)}$= $C^{(i)}$ = constant, $J^{(i)} \sim \dfrac{\alpha_s(\mu_0)}{x\, k_1^+\, k_2^+}$

so get
$$A_{00} \propto C^{(i)} \int \frac{S^{(i)}(k_1^+, k_2^+)}{k_1^+\, k_2^+} \int \frac{\phi_\pi(x)}{x} = s^{\rm eff}(\mu_0)\, \langle x^{-1}\rangle_\pi$$

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- If $\langle x^{-1}\rangle_\pi \simeq \langle x^{-1}\rangle_\rho$ then $|r^{D\pi}| = |r^{D\rho}|$

$$|r^{D\pi}| = \frac{|A(\bar{B}^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|} = 0.77 \pm 0.05\,, \qquad |r^{D\rho}| = 0.80 \pm 0.09$$
$$|r^{D^*\pi\pi\pi\pi}| = 0.98 \pm 0.27$$

- Also would predict that $\delta^\rho = \delta^\pi$
- If we fit the complex s^{eff}=$(428 \pm 48 \pm 100\,\text{MeV})\exp(i(44^\circ \pm 7^\circ))$

ie natural size, $s^{\text{eff}} \simeq \Lambda_{\text{QCD}}$ from dim. analysis

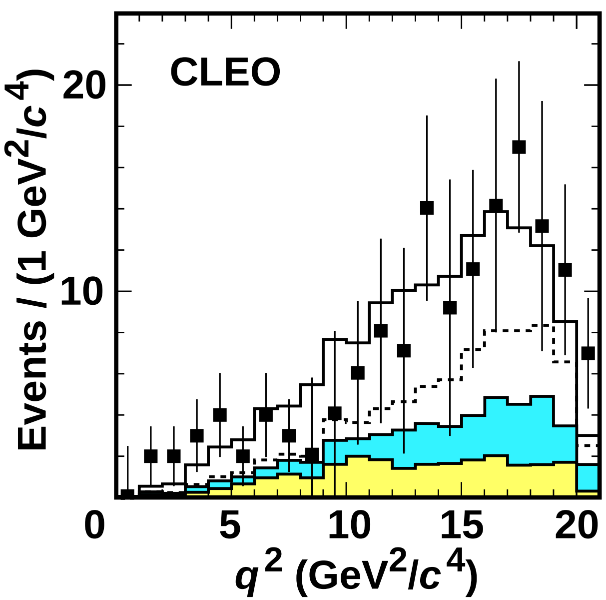
More Predictions

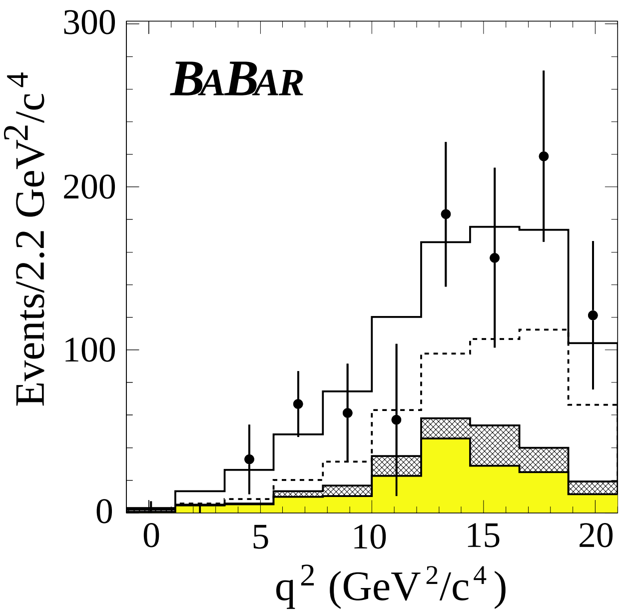
naive factorization for color suppressed decays $\longrightarrow$  ?

Heavy-to-Light Decays

- Large q^2 accessible on the Lattice ($B \to \pi\ell\nu$, $q^2 \gtrsim 15\,\mathrm{GeV}^2$)
- For small q^2, $E \gg \Lambda_{\mathrm{QCD}}$ and large energy factorization applies

$B \to \rho\ell\nu$





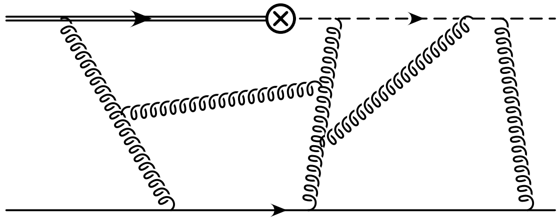
Why is it interesting?

- Important ingredient for $B \to \pi\pi, \pi\rho, \rho\rho$ (CP violation)
- Phenomenology: $|V_{ub}|$, $B \to \rho\gamma$, $B \to K^* e^+ e^-$, $\ldots$

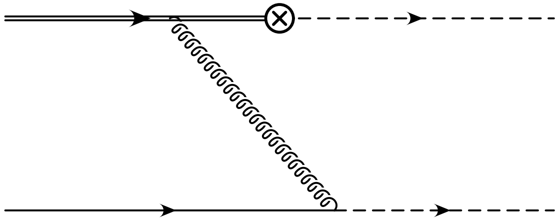
Heavy-to-Light Form Factors

pseudoscalar: f_+, f_0, f_T , vector: $V, A_0, A_1, A_2, T_1, T_2, T_3$

"Soft part"



"Hard part"



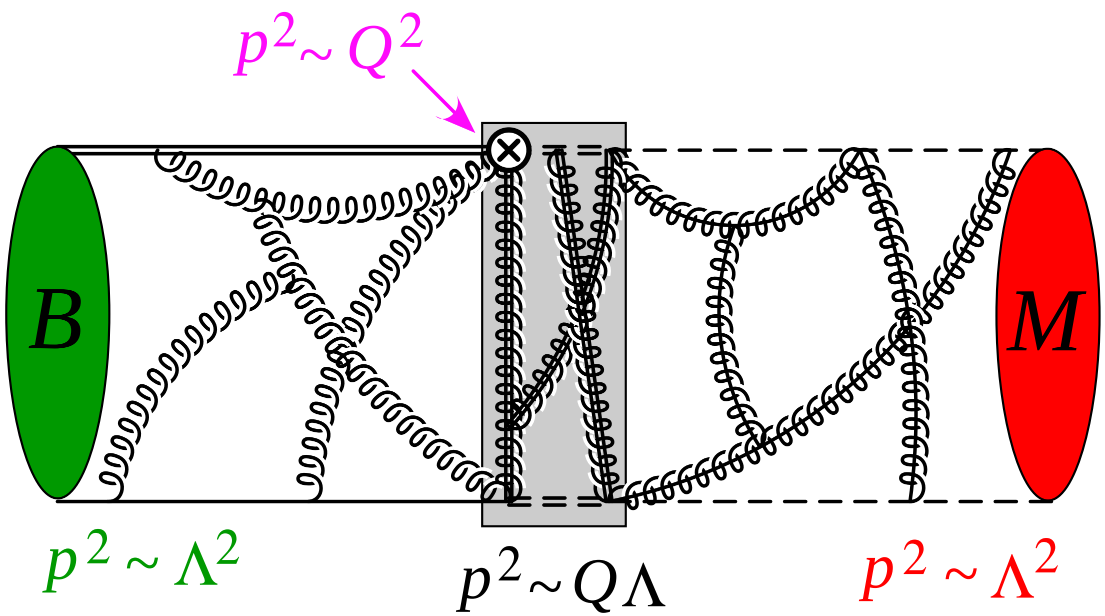
Form Factor Results

Bauer, Pirjol, I.S.
Beneke, Feldmann

Our result $f(Q) = f^F(Q) + f^{NF}(Q)$

$$f^F(Q) = \frac{f_B f_M m_B}{4E^2} \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z, E, m_b, \mu_0) \times J(z, x, r_+, E) \phi_M(x) \phi_B^+(r_+)$$

$$f^{\rm NF}(Q) = C_k(E, m_b) \, \zeta_k(Q\Lambda, \Lambda^2) \, .$$



result at LO in λ, all orders in α_s, where $Q = \{m_b, E_M\}$

Results

Pirjol, I.S.

B to pseudoscalar

$$
\begin{aligned}
f_+ &= T_\zeta^{f_+}\,\zeta^P + N_0 \int\! d\mathcal{M} \Big[T_a^{f_+}\, J_a(x,r_+) + T_b^{f_+}(z)\, J_b(z,x,r_+) \Big] \phi_P(x)\, \phi_B^+(r_+)\,, \\
\frac{m_B}{2E} f_0 &= T_\zeta^{f_0}\,\zeta^P + N_0 \int\! d\mathcal{M} \Big[T_a^{f_0}\, J_a(x,r_+) \;+ T_b^{f_0}(z) J_b(z,x,r_+) \Big] \phi_P(x)\, \phi_B^+(r_+)\,, \\
f_T &= T_\zeta^{f_T}\,\zeta^P + N_0 \int\! d\mathcal{M} \Big[T_a^{f_T}\, J_a(x,r_+) + T_b^{f_T}(z) J_b(z,x,r_+) \Big] \phi_P(x)\, \phi_B^+(r_+),
\end{aligned}
$$

Results

Pirjol, I.S.

B to vector

$$
\begin{aligned}
V &= T_\zeta^V \zeta_\perp^V + N_\perp \int d\mathcal{M}\Big[T_a^V J_a^\perp(x,r_+) + T_b^V(z) J_b^\perp(z,x,r_+)\Big]\phi_B^+(r_+)\phi_\perp^V(x)\,, \\
A_0 &= T_\zeta^{A_0} \zeta_\parallel^V + N_\parallel \int d\mathcal{M}\Big[T_a^{A_0} J_a(x,r_+) + T_b^{A_0}(z) J_b(z,x,r_+)\Big]\phi_B^+(r_+)\phi_\parallel^V(x)\,, \\
\frac{m_B}{2E} A_1 &= T_\zeta^{A_1} \zeta_\perp^V + N_\perp \int d\mathcal{M}\Big[T_a^{A_1} J_a^\perp(x,r_+) + T_b^{A_1}(z) J_b^\perp(z,x,r_+)\Big]\phi_B^+(r_+)\phi_\perp^V(x) \\
A_2 - \frac{m_B}{2E} A_1 &= \frac{m_V}{E} T_\zeta^{A_{12}} \zeta_\parallel^V + \frac{m_V}{E} N_\parallel \int d\mathcal{M}\Big[T_a^{A_{12}} J_a(x,r_+) \\
&\quad + T_b^{A_{12}} J_b(z,x,r_+)\Big]\phi_B^+(l_+)\phi_\parallel^V(x)\,, \\
T_1 &= T_\zeta^{T_1} \zeta_\perp^V + N_\perp \int d\mathcal{M}\Big[T_a^{T_1} J_a^\perp(x,r_+) + T_b^{T_1}(z) J_b^\perp(z,x,r_+)\Big]\phi_B^+(l_+)\phi_\perp^V(x)\,, \\
\frac{m_B}{2E} T_2 &= T_\zeta^{T_2} \zeta_\perp^V + N_\perp \int d\mathcal{M}\Big[T_a^{T_2} J_a^\perp(x,r_+) + T_b^{T_2}(z) J_b^\perp(z,x,r_+)\Big]\phi_B^+(l_+)\phi_\perp^V(x)\,, \\
T_3 - \frac{m_B}{2E} T_2 &= \frac{m_V}{E} T_\zeta^{T_{23}} \zeta_\parallel^V + N_\parallel \frac{m_V}{E} \int d\mathcal{M}\Big[T_a^{T_{23}} J_a(x,r_+) \\
&\quad + T_b^{T_{23}}(z) J_b(z,x,r_+)\Big]\phi_B^+(l_+)\phi_\parallel^V(x)\,,
\end{aligned}
$$

Implications

- $V = m_B A_1/(2E)$ and $T_1 = m_B T_2/(2E)$ by helicity symmetry
  Burdman, Hiller

- certain $A_{1,2}$ and $T_{1,2}$ combinations are m_V/E suppressed
- goal is to identify processes besides $B \to \pi\ell\nu$ that depend on same non-perturbative parameters, egs. $B \to \pi\pi$, $B \to \pi\ell^+\ell^-$, $B \to \gamma\ell\nu$
- If lattice can get points in the low q^2 region they can read off important hadronic moments by fitting certain linear combinations

eg. $$\frac{V}{T_\zeta^V} - \frac{T_1}{T_\zeta^{T_1}} \propto \mathcal{T}(E) \frac{\langle x^{-1}\rangle_V \langle r_+^{-1}\rangle_B}{E^2}$$

- For $B \to \pi\pi$ SCET reduces the lattice problem to
  $(B \to \pi)\times(0 \to \pi)$

  Maybe in the future we can get to
  $(B \to 0)\times\ (0 \to \pi)\times\ (0 \to \pi)$

Form Factor Result

More comments

$$f^F(Q) = \frac{f_B f_M}{Q^2}\int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z,Q,\mu_0) \times J(z,x,r_+,Q,\mu_0,\mu)\phi_M(x,\mu)\phi_B^+(r_+,\mu)$$

$$f^{\rm NF}(Q) = C_k(Q,\mu)\,\zeta_k(Q\Lambda,\Lambda^2,\mu)\,.$$

- No suppression of f^F/f^{NF} by an $\alpha_s(\mu_0)$ is observed, might expect $f^F(0) \sim f^{NF}(0) \simeq (\Lambda/E_\pi)^{3/2} \sim 0.08$
- In $B \to \pi\pi$ BBNS use $f^{NF} \gg f^F$
  Keum, Li, Sanda use "$f^{NF} \ll f^F$" (with a different definition)
- Fit with f.f. models gives, $f_+(0) = 0.23 \pm 0.04$ Luo, Rosner
- Data for $f_+(q^2=0)$, $V(q^2=0)$, $T_1(q^2=0)$, $\ldots$, will eventually tell us how f^F compares with f^{NF}
- Theory Corrections are $\Lambda/E_\pi \sim 20-30\%$ to this factorization, growing as E_π gets smaller

$B \to MM$

eg. Measure $\sin(2\alpha)$ with $B^0(t) \to \pi^+\pi^-$, $\bar{B}^0(t) \to \pi^+\pi^-$

$$\mathcal{A}_{CP}(t) = -S_{\pi\pi} \sin(\Delta m_B t) + C_{\pi\pi} \cos(\Delta m_B t)$$

$$S_{\pi\pi} = \frac{2\,\mathrm{Im}\lambda}{1+|\lambda|^2}, \quad C_{\pi\pi} = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \quad \lambda = e^{2i\alpha} \frac{1+e^{i\gamma} P/T}{1+e^{-i\gamma} P/T}$$

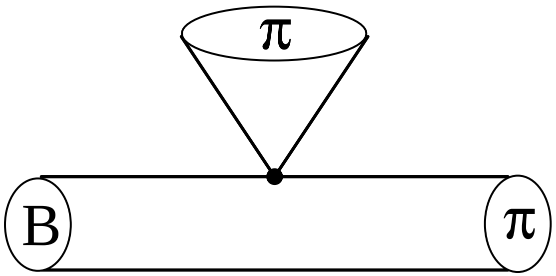
T = tree **P** = penguin

P/T$\neq 0$, need information from QCD (or isospin analysis)

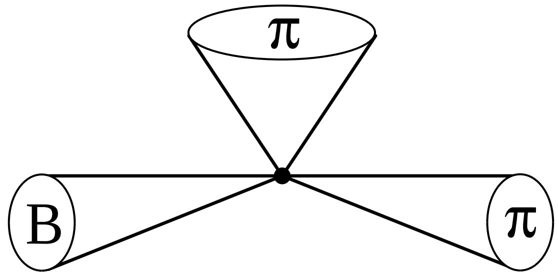
$B \to \pi\pi$

In “QCD Factorization”

Beneke, Buchalla, Neubert, Sachrajda



$F^{B\to\pi}$, $\phi_\pi(x)$

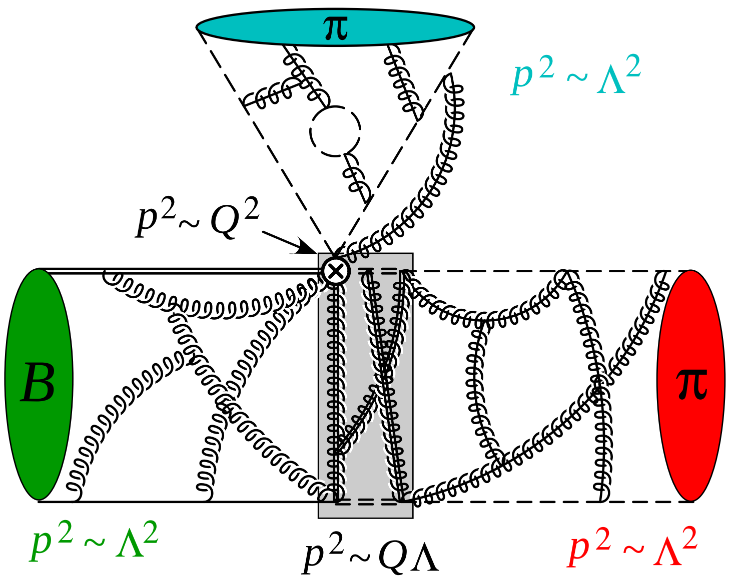


$\phi_B(\xi), \phi_\pi(x), \phi_\pi(y)$

In SCET

Chay, Kim

Bauer, Pirjol, Rothstein, I.S. (in progress)



involves

ζ_π, ϕ_B, $\phi_\pi(x)$

Issues in $B \to \pi\pi$

1) Factorization/Exponentiation of gluons beyond $\mathcal{O}(\alpha_s)$ Chay, Kim

2) Result if $\alpha_s(Q\Lambda \sim 1.1 - 1.6\,\mathrm{GeV})$ is not perturbative?

3) New Soft-Collinear messenger modes Becher,Hill,Neubert

   Could spoil factorization in $B \to \pi\pi$, . . ., etc.

4) Glauber Gluons beyond $\mathcal{O}(\alpha_s)$ (like Coulombic exchange)

5) Numerical stability and convergence, chirally enhanced terms?

6) Error estimates, could power corrections dominate $B^0 \to \pi^0\pi^0$?
   (since BBNS and pQCD disagree with new Belle and BaBar data)
   or is something else going on . . . ?

Workshop Issues

- Is it always true that the vacuum factorizes? Manohar

  $|0\rangle = |0\rangle_c \otimes |0\rangle_{us}$

- Can SCET help to explain the cross section for $e^+e^- \to J/\Psi X$, $e^+e^- \to \eta_c J/Psi$? Fleming

- Could singularities forbid factorization below the $Q\Lambda$ scale? Feldmann

- Claim of a non-zero time ordered product in $\mathrm{SCET_{II}}$ for B-decays. Pirjol

- Claim soft-Collinear modes exist and spoil factorization in some cases. Neubert

- A proposal for a new choice of fields for $\mathrm{SCET_I}$, and doubts about soft-collinear modes. Chay

Outlook

- SCET gives operator definitions to universal hadronic parameters $\to$ need to measure these with experiment
- Subfields: Jet Physics, B Physics, $b\bar{b}$ Physics
- Allows power corrections to be addressed in a model independent way (even when we lack a rigorous OPE)
- Need to carefully examine expansion for each process and improve our understanding of power corrections to go beyond 20-30% accuracy for B's
- SCET applies to many inclusive/exclusive processes
  A lot of theory and phenomenology left to study . . .

Colors

This is blue

This is red

This is brown

This is magenta

This is Dark Green

This is Dark Blue

This is Green

This is Cyan

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks

Test how this color looks