Theory of Radiative & Rare Decays

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- Introduction [*Why are we interested in rare decays?*]
- Inclusive FCNC *B* decays [*General properties*]
    - $B \to X_s \gamma$
    - $B \to X_s l^+l^-$
- Exclusive channels [*Generalities & a few selected examples*]
    - $B \to l^+l^-$
- Conclusions

Introduction

Why are we interested in rare decays ?

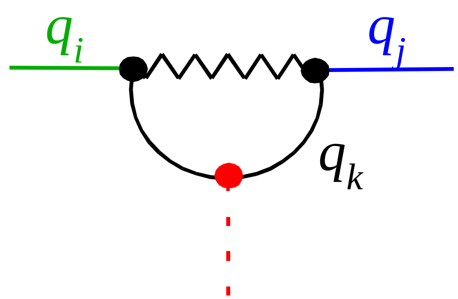
Rare processes are interesting when their suppression is associated to some (hopefully broken...) conservation law [e.g.: $\not{B} \Leftrightarrow p$ decay, $\not{L} \Leftrightarrow 2\beta 0\nu$ decay, ...]

Flavor Changing Neutral Currents

[especially ~~CP~~–FCNC]

$$q_i \rightarrow q_j + \gamma,\ l^+ l^-, \bar{\nu}\nu$$

are the ideal candidates to study in detail the breaking of the (approximate) *flavor symmetry* of the SM

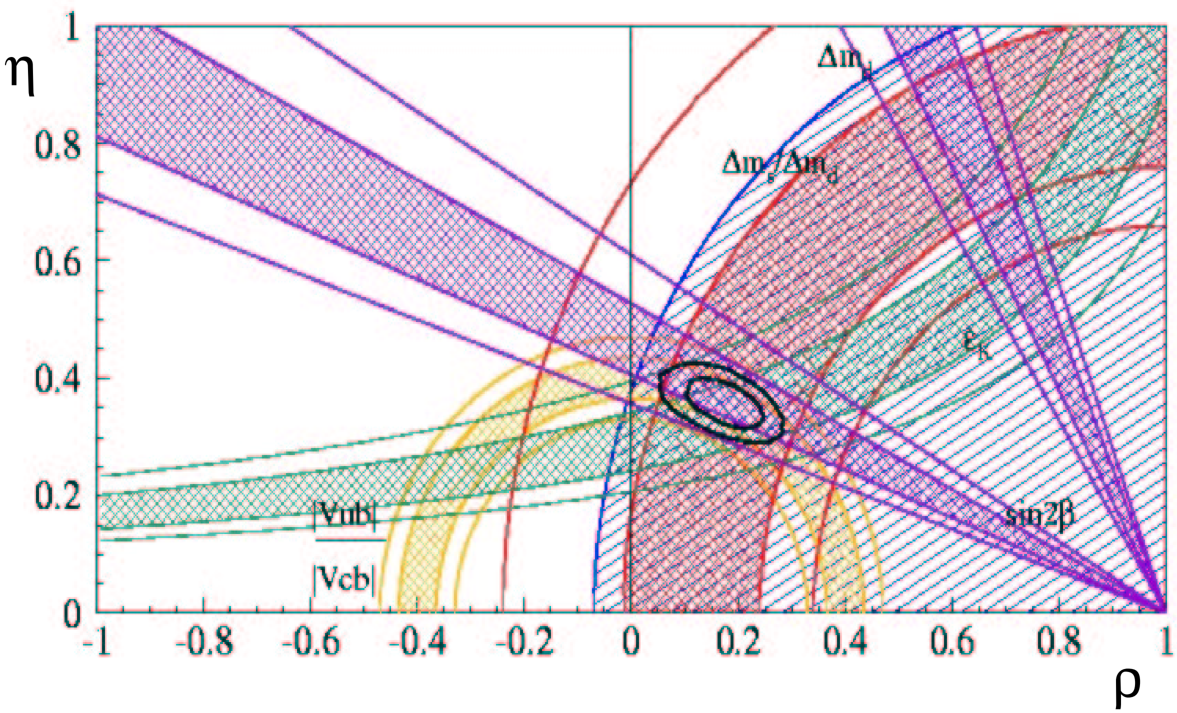


- no tree−level contributions within the SM
- likely to be dominated by short−distance dynamics [*key point*]

precise indirect determ. of flavor mixing within the SM [e.g.: V_{td}]

enhanced sensitivity to possible new degrees of freedom

Available data on $\Delta F=2$ FCNC amplitudes (meson−antimeson mixing) already provides serious constraints on the scale of New Physics:



e.g.: $K^0-\bar{K}^0$ mixing

$\Downarrow$

$\Lambda \gtrsim 100$ TeV

for $O^{(6)} \sim \dfrac{(\bar{s}d)^2}{\Lambda^2}$

much more severe than bounds on the scale of flavor−conserving operators from e.w. precision data

...while a natural stabilization of the Higgs potential $\Rightarrow$ $\Lambda \sim 1$ TeV

After the recent precise data from *B* factories, it is more difficult [although not impossible] to believe that this is an accident

The Flavor Problem

Two possible solutions:

- *pessimistic* [very unnatural]: $\Lambda > 100$ TeV
  $\Rightarrow$ almost nothing to learn from other FCNC processes
  (but also very difficult to find evidences of NP at LHC...)

- *natural*: $\Lambda \sim 1$ TeV + flavor–mixing protected by additional symmetries
  $\Rightarrow$ still a lot to learn from *rare decays*

- Present fit of the CKM unitarity triangle involve only two types of amplitudes sensitive to NP: *K–K* mixing and *B–B* mixing ($\Delta F=2$ transitions only) $\Rightarrow$ we known very little yet about $\Delta F=1$ transitions

- Present CKM fits provide only a consistency check of the SM hypothesis but do not provide a bound on the NP parameter space $\Rightarrow$ only with the help of rare decays we can study the underlying flavor symmetry in a model−independent way

FCNC B decays

General properties:

On general grounds, the *inclusive* transitions $B \to X_{(s,d)}\gamma$ & $B \to X_{(s,d)}l^+l^-$ [and eventually $B \to X_{(s,d)}\nu\nu$] are the best candidates to perform *precision* tests of flavor dynamics:

- Precise (NLO & NNLO) calculations of the inclusive decay rates within perturbative QCD ($m_b \gg \Lambda_{QCD}$)

$$\Gamma(b \to s\gamma) \xrightarrow{m_b \to \infty} \Gamma(B \to X_s\gamma)$$

- Systematic control of the (suppressed) non−perturbative corrections via the heavy quark expansion

$O(\Lambda_{QCD}/m_b)$ corrections

well under control (errors < 5%) in ***sufficiently inclusive*** observables

long−distance contributions
$b \to s\,(c\bar{c}) \to s\,(\gamma, l^+l^-)$

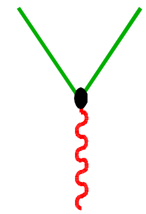
under control (in the charm case) *far from the resonance region*
$\Rightarrow O(\Lambda_{QCD}/m_c)$

The perturbative calculation:

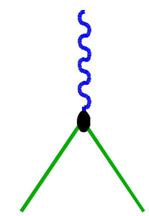
$$H_W = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_i C_i(\mu) Q_i$$

Effective operators
sensitive to short distances:

$$Q_7 = \frac{e^2 m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$$



$$Q_8 = \frac{g_s^2 m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} \tau^a b_R G^a_{\mu\nu}$$

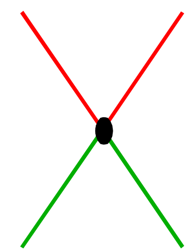


+

ordinary 4−quark
operators $[Q_{1-6}]$

$$Q_9 = \frac{e^2}{16\pi^2} \bar{s}_L \gamma^\mu b_L \bar{l} \gamma_\mu l$$

$$Q_{10} = \frac{e^2}{16\pi^2} \bar{s}_L \gamma^\mu b_L \bar{l} \gamma_5 \gamma_\mu l$$



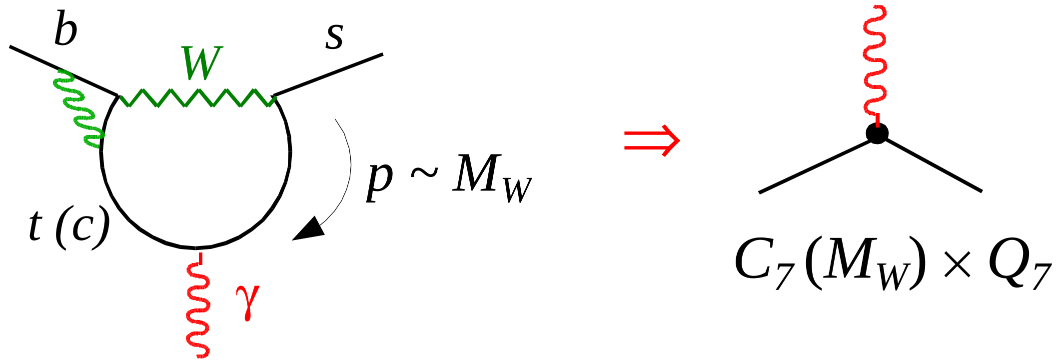
$$A(B \to f) = \sum_i C_i(\mu) \langle f | Q_i | B \rangle (\mu)$$

A consistent N(N)LO analysis [$\alpha_S^{N+1} \ln(m_b/M_W)^N$] requires 3 steps:

N(N)LO $C_i(M_W)$ + N(N)LO RGE + N(N)LO matrix elements

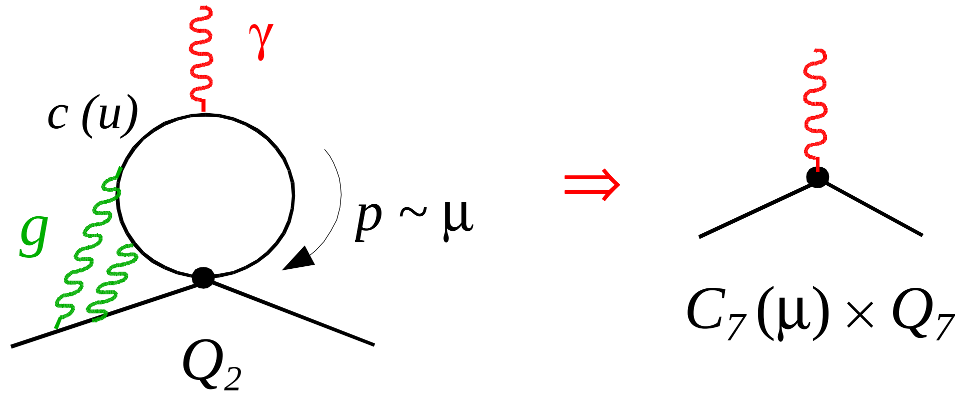
I. The initial conditions

- sensitivity to short-distances [NP]



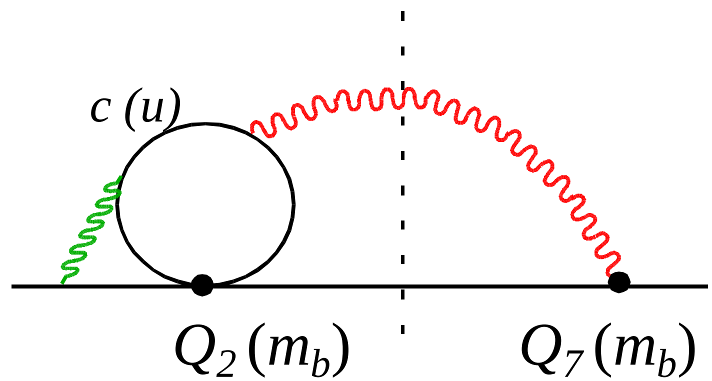
II. The RGE evolution

- QCD *dilution* of the s.d. sensitivity [resummation of large logs: $\alpha_S \ln(\mu/M_W) \sim O(1)$]



III. The matrix elements

- sensitivity to long−distances ($c\bar{c}$ threshold, m_c dependence,...)



N.B.: operators such as Q_{10} [*axial current ~ Z penguin*], not contaminated by the mixing with 4−quarks, are particularly clean probes of s.d. dynamics $\Rightarrow B \to X_s l^+ l^-$

• $B \to X_s \gamma$

"*The most effective NP killer*"

NLO enterprise completed already a few years ago,
all steps recently cross−checked:

I. $C_i(M_W)$ [$C_{7,8}$ @ 2 loops] Adel & Yao '94 + several checks (also beyond SM)

II. RGE [$Q_{7,8} \leftrightarrow Q_{1-6}$ @ 3 loops] Chetyrkin, Misiak, Munz '97
+ Gambino, Gorban, Haisch, 03

III. $\langle Q_i \rangle$ [Q_{1-6} @ 2 loops] Greub, Hurth, Wyler '96 + Buras *et al*. '01

⬇

Residual scale dependence in the BR ~ 4% !

- At this level of accuracy also subleading electroweak corrections become relevant [main effect: running of α_{em}] Czarnecki & Marciano '98; Gambino & Haisch '00

- Largest uncertainty induced by charm mass dependence (III.) : 10% shift in BR for $m_c^{\text{pole}} \to m_c^{\text{MS}}(\mu)$ [NNLO effect] Misiak, Gambino '01

- Non−perturbative $1/m_{b,c}$ corrections well under control in the *total rate*:

  no linear terms;

  small $(\Lambda_{QCD}/m_{b,c})^2$ terms (~ 2−3 %) known from $\Gamma(B \to X_c\, l\, \nu)$ & $(M_{B^*} - M_B)$

  [HQET]

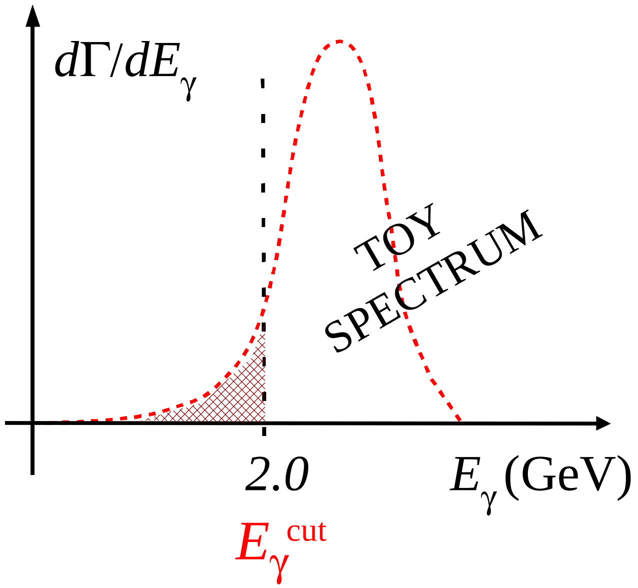
- The most serious problem is the fact that the fully inclusive rate *is not accessible*:

  extrapolation below E_γ^{cut}

The E_γ spectrum [*shape function*]

need to be determined from data

[*effective* Λ_{QCD}/m_b *corrections*]



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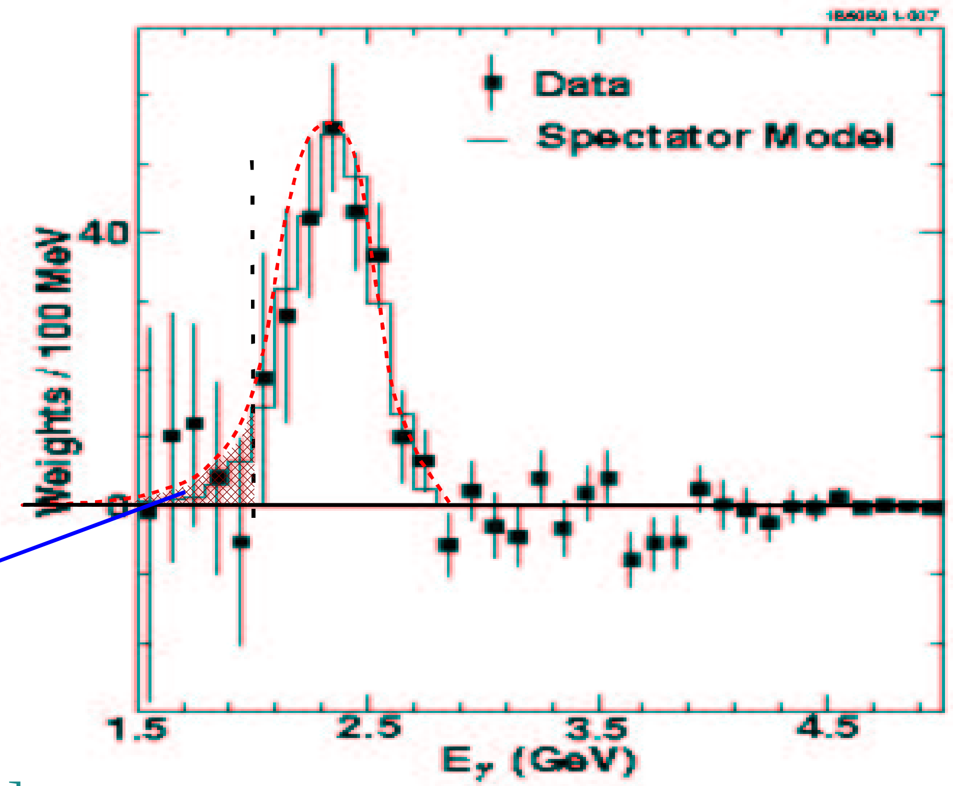

Error in the extrapolation ~ 5%

[CLEO '01 + Kagan, Neubert; Ali, Greub]

Putting all the ingredients together:

Most recent SM th. estimate:

$$B(B \to X_s \gamma) = (3.73 \pm 0.30) \times 10^{-4}$$

[Misiak, Gambino, 01]

- partial inclusion of NNLO terms
  [$m_c^{\text{pole}} \to m_c(\mu)$]
- error estimate includes an educated guess on NNLO terms

To be compared with:

$(3.21 \pm 0.43 \pm 0.27\,^{+0.18}_{-0.10}) \times 10^{-4}$ CLEO '01

$(3.36 \pm 0.53 \pm 0.42\,^{+0.50}_{-0.54}) \times 10^{-4}$ BELLE '01

$(3.88 \pm 0.36 \pm 0.37\,^{+0.43}_{-0.23}) \times 10^{-4}$ BABAR '02

$(3.34 \pm 0.38) \times 10^{-4}$ W.A.

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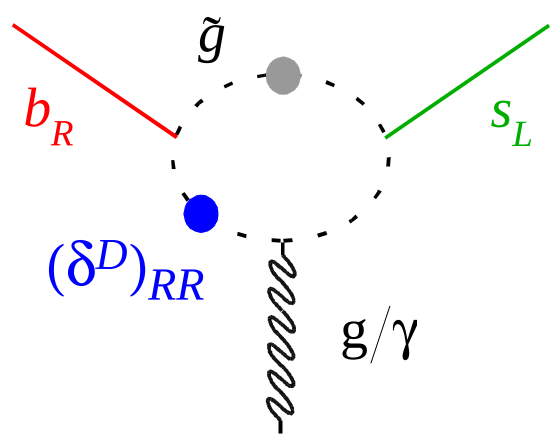
A great success for the SM... <u>and a lot of problems for many of its extensions !</u>

E.g.: strong constraints on the SUSY mixing terms which could induce $A_{CP}(\phi K_S) \neq A_{CP}(\psi K_S)$

In my opinion $A_{CP}(\phi K_S) < 0$ requires a rather ugly conspiracy



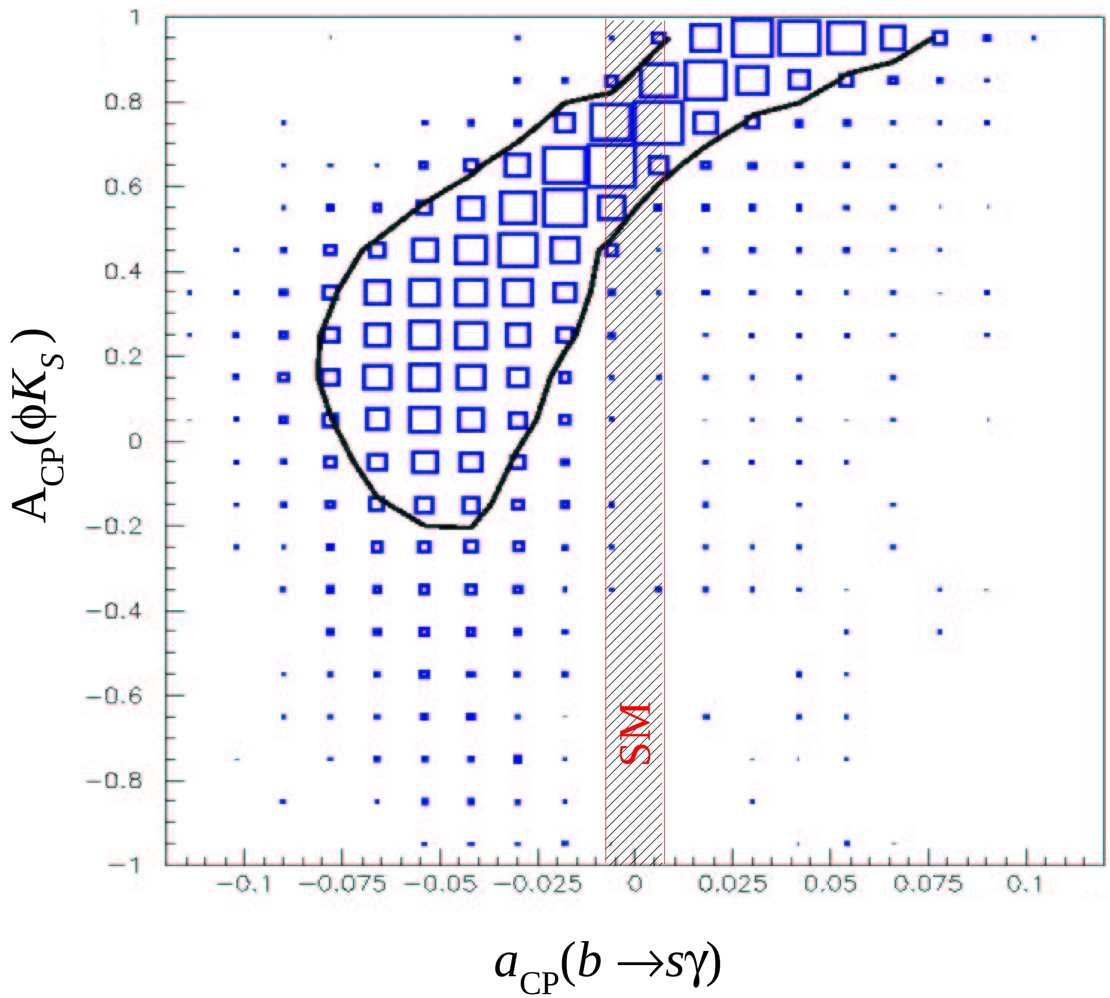
Masiero & Murayama + many others...

Several th. collaborations started to analyse the missing pieces necessary to predict $B(B \to X_s \gamma)$ at NNLO within the SM [Misiak & Co.] $\Rightarrow$ *long & challenging project...*

Beside the rate, very interesting short−distance info can also be extracted from the inclusive CP asymmetry:

$$a_{CP} = \frac{\Gamma(\bar{B}) - \Gamma(B)}{\Gamma(\bar{B}) + \Gamma(B)} \overset{\text{(SM)}}{\approx} 0.6\%$$

Kagan, Neubert '98



Ciuchini *et al.* '03

Suppressed within SM by the smalness of $\Im(V_{tb}^* V_{ts})$ possible large effects (~10%) with new CPV phases

Present exp. bounds ~ 10%

$\Rightarrow$ still 1 order of magnitude of possible NP contributions to be explored

• $B \to X_s l^+ l^-$

- Different LL count. than in $B \to X_s \gamma$ [$Q_9 \leftrightarrow Q_{1-6}$ starts @ 1 loop $\Rightarrow$ NNLO simpler]
- Sensitivity to e.w. box & Z penguins not present in $B \to X_s \gamma$ [Q_9 & Q_{10}]
- Dangerous long−distance contamination from real $c\bar{c}$ states
  [$\langle Q_i \rangle$ & non−pert. effects more complicated than in $B \to X_s \gamma$]

Very recently full NNLO analyses available for both dilepton spectrum & lepton FB asymmetry:

I. $C_i(M_W)$ Bobeth, Misiak & Urban, '00;
II. RGE Gambino, Gorban, Haisch, '03;
III. $\langle Q_i \rangle$ Asatryan, Asatrian, Greub, Walker '01−'02;
Ghinculov, Hurh, G.I. & Yao, '02−'03;

⬇

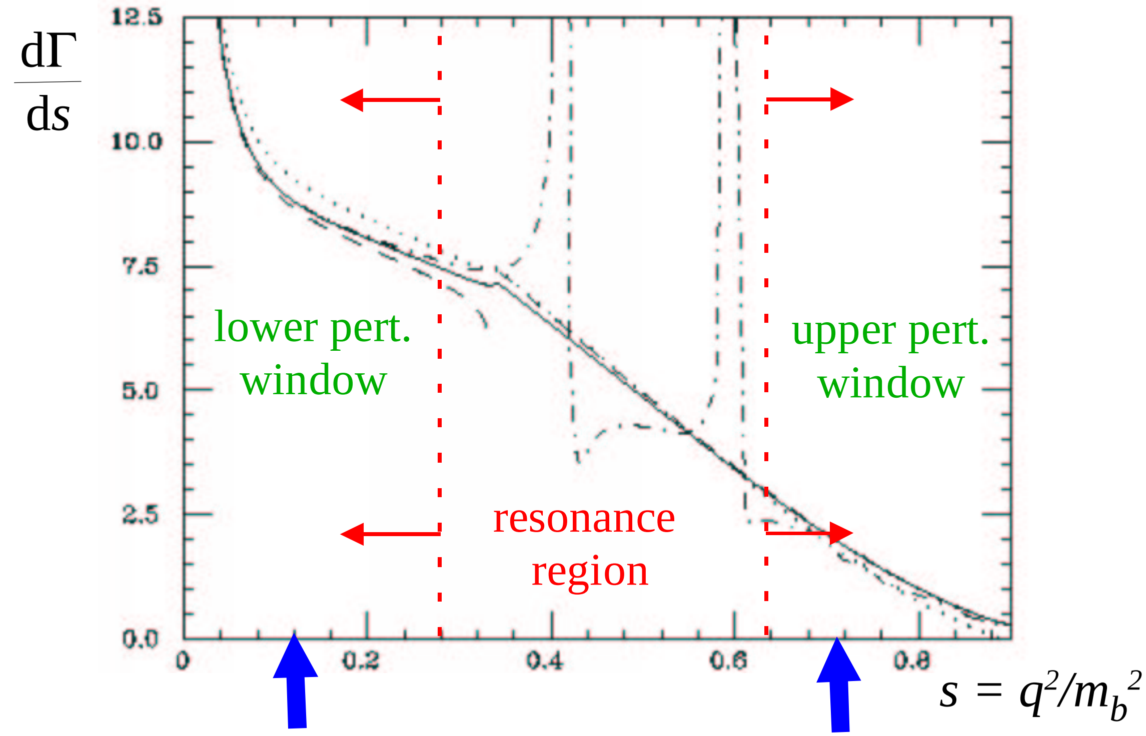
Residual scale dependence in the dilepton spectrum: from 3% to 7% (depending on the kin. region); even smaller for the FB asymmetry

The dilepton spectrum

We can define two clean *perturbative windows* free from large non−pert. effects:

The two regions are affected by different th. (systematic) errors and probe different s.d. structures

It would be very useful to *quote separately* the measurements of the BR in these two regions



larger rate
sens. to $Q_7 \times Q_9$
small $1/m_b$ corr.

larger $M_{had.}$ cuts
larger charm corr.
larger scale dep.

more sens. to Q_{10}
small scale dependence
small charm corr.
small $M_{had.}$ cuts

larger $1/m_b$ corr.
low rate

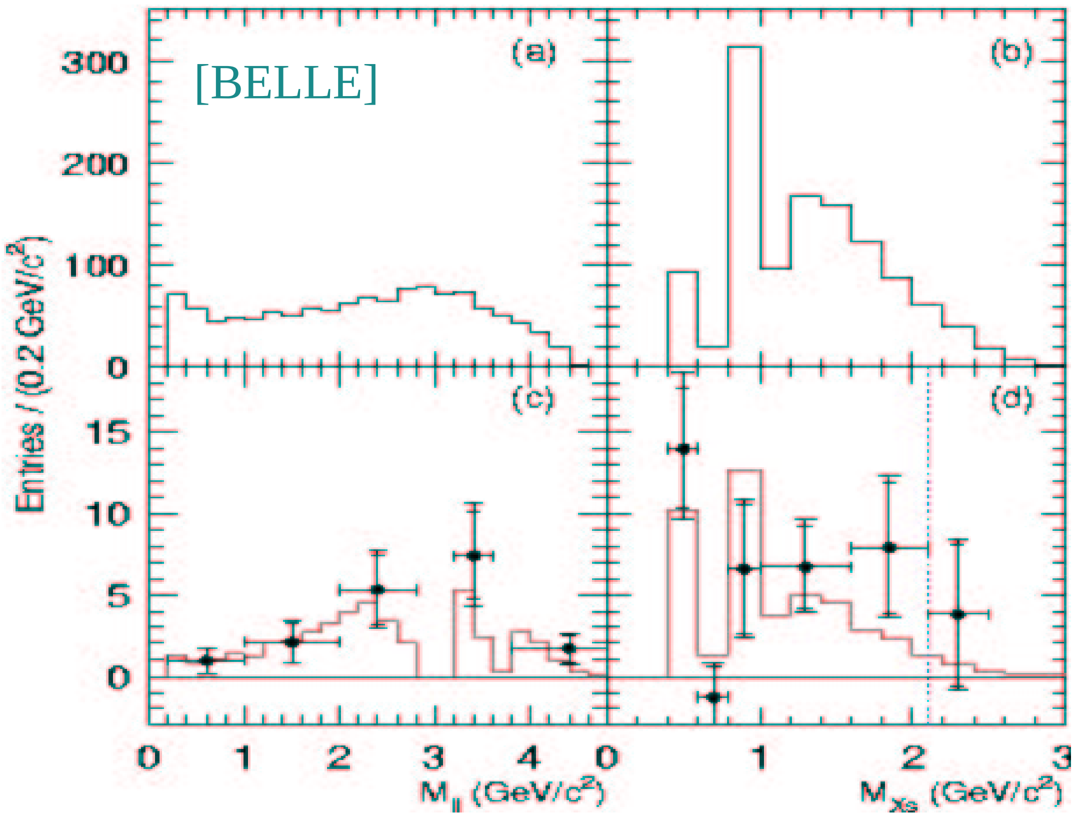
NNLO SM predictions:

$$B(B \to X_s l^+ l^-;\ q^2 \in [1,6]\ \mathrm{GeV}^2) = (1.60 \pm 0.19) \times 10^{-6}$$

$$B(B \to X_s l^+ l^-;\ q^2 > 14.4\ \mathrm{GeV}^2) = (3.84 \pm 0.75) \times 10^{-7}$$

Ghinculov, Hurh, G.I. & Yao, '02−'03;

This summer B factories have reached the 5σ level (discovery level) on the combined (l=e,μ) (semi−) inclusive branching ratios:



$(6.1 \pm 1.4\,^{+1.4}_{-1.1})\times 10^{-6}$ BELLE '03

$(6.3 \pm 1.6\,^{+1.8}_{-1.5})\times 10^{-6}$ BABAR '03

$(6.2 \pm 1.1\,^{+1.6}_{-1.3})\times 10^{-6}$ W.A.

Extrapolated result, to be compared with $B(B \to X_s l^+ l^-)^{\rm SM} = (4.2 \pm 0.7)\times 10^{-6}$

Ali et al. '02;

⇒ Promising prospects for the future! ⇐

N.B.: another interesting candidate for a large $A_{CP}(\phi K_S) \neq A_{CP}(\psi K_S)$ namely a non−standard $b \to s$ Z−penguin [G. Hiller *et al.*] is already strongly constrained by these data [$A_{CP}(\phi K_S) < 0$ excluded]

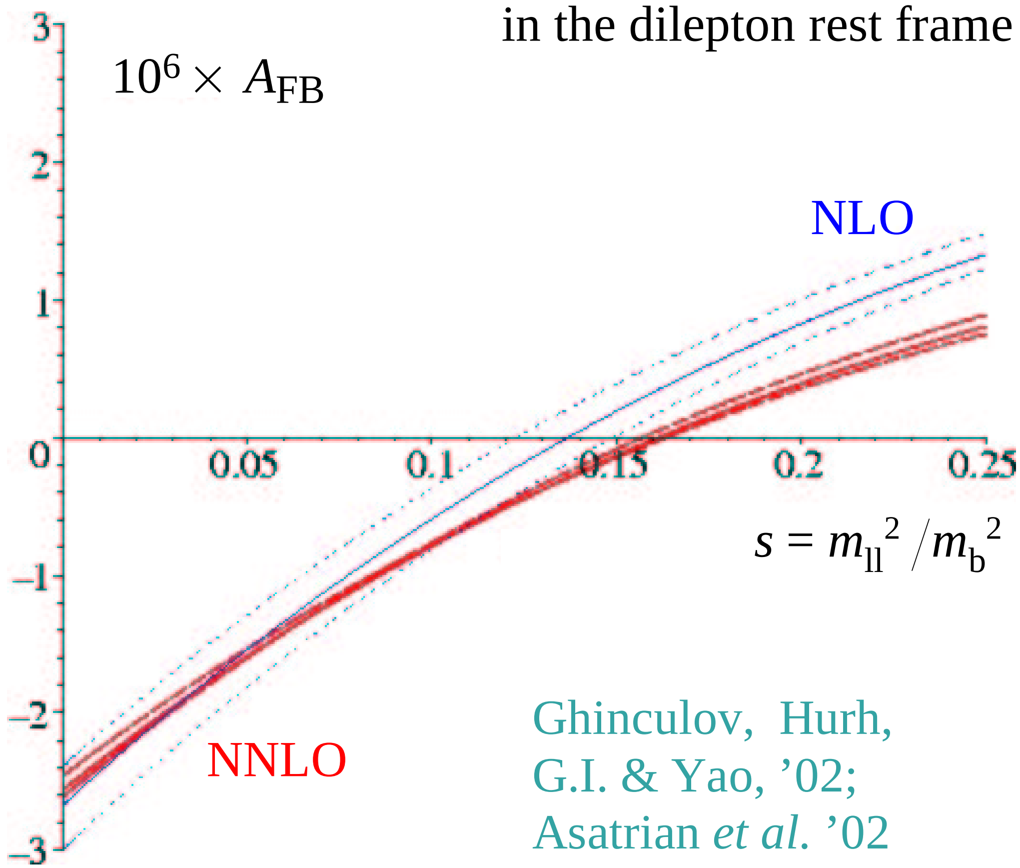
The lepton FB asymmetry

Probably the most interesting observable in $B \to X_s l^+ l^-$ decays:

$$A_{FB} = \int \frac{d^2 B(B \to X_s \mu^+ \mu^-)}{ds\, d\cos\vartheta} sgn(\cos\vartheta) \quad \propto \quad \Re\left[C_{10}^* \left(s C_9^{eff}(s) + r(s) C_7\right)\right]$$

th. errror $\lesssim 5$ %

θ = angle between μ^+ & B momenta in the dilepton rest frame



Ghinculov, Hurh, G.I. & Yao, '02; Asatrian *et al.* '02

- direct access to the *relative phases* of the C_i
- proportional to C_{10} (interf. of axial & vector currents) $\Rightarrow$ small QCD corrections

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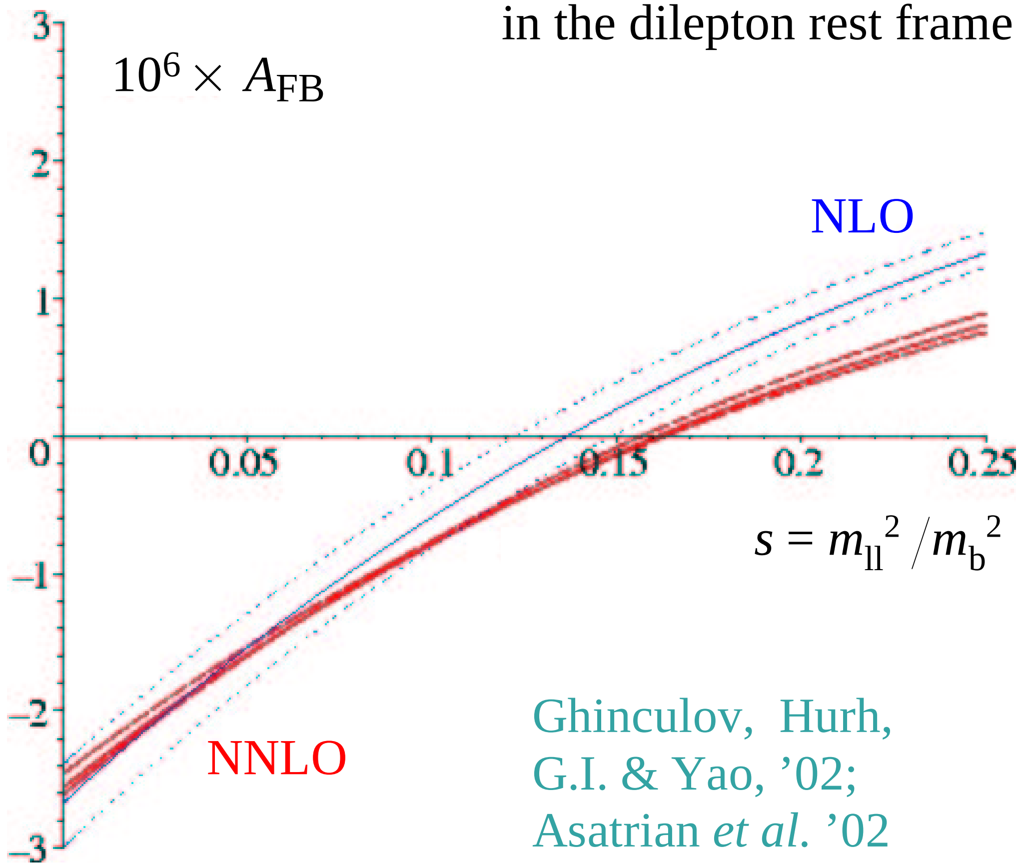
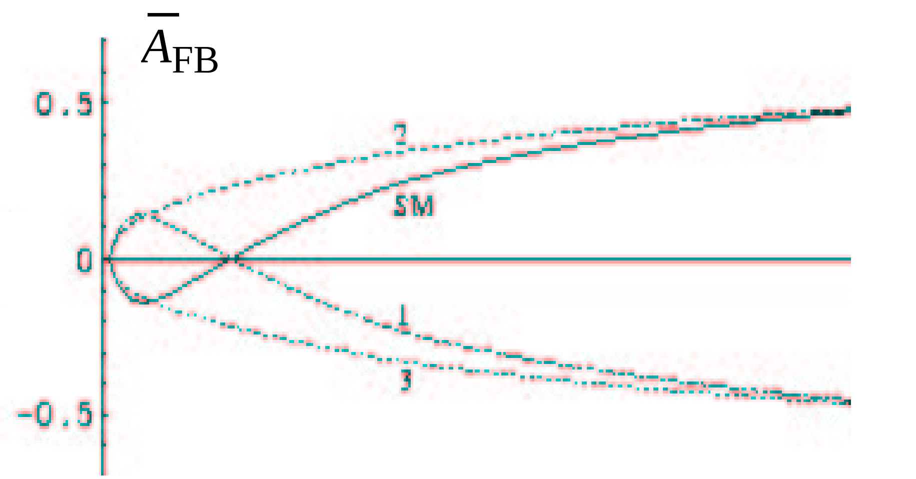


Ghinculov, Hurh, G.I. & Yao, '02; Asatrian *et al.* '02

- a very useful probe of non−standard scenarios:



Ali *et al.* '01

Exclusive FCNC B decays

The accuracy on *exclusive* FCNC B decays of the type $B \to H + (\gamma, l^+ l^-)$ depends on the th. control of $B \to H$ *hadronic form factors.*

$\Rightarrow$ several progress in the last few years [HQS, SCET $\Leftrightarrow$ LCSR, Lattice] but typical errors still ~ 30%

The most difficult exclusive observables are the total branching ratios

$\Rightarrow$ the s.d. info which we can extract from the latest data on $\mathrm{B}(B \to X_s l^+ l^-)$ is already superior to what we could get from $\mathrm{B}(B \to K^* l^+ l^-)$ & $\mathrm{B}(B \to K\, l^+ l^-)$

However, *f.f.* uncertainties can be considerably reduced in appropriate ratios or differential distributions

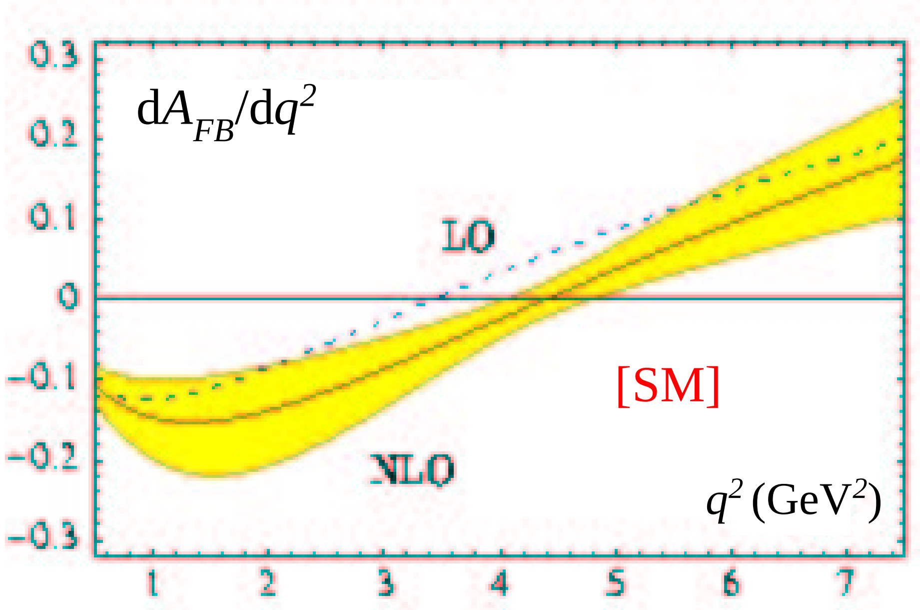
$\Rightarrow$ especially interesting when the corresponding inclusive observable is not exp. accessible, e.g:

$$\bar{A}_{\mathrm{FB}}\,(B \to K^* l^+ l^-) \qquad R(\rho\gamma/K^*\gamma) = \frac{\mathrm{B}(B \to \rho\gamma)}{\mathrm{B}(B \to K^*\gamma)}$$

A) Properties of $A_{FB}(s)$ indep. from the detailed structure of the form factors:

- $A_{FB}(s) = 0$ for $s = q^2/m_b^2 \sim C_7/C_9$

  Burdman '98; Ali *et al.* '00;
  Beneke, Feldmann, Seidel '01 →



- Within the SM $A_{FB}^{(\bar{B})}(s) < 0$ for $s < s_0$ & $A_{FB}^{(\bar{B})}(s) = -A_{FB}^{(B)}(s)$ (modified by new CPV phases)

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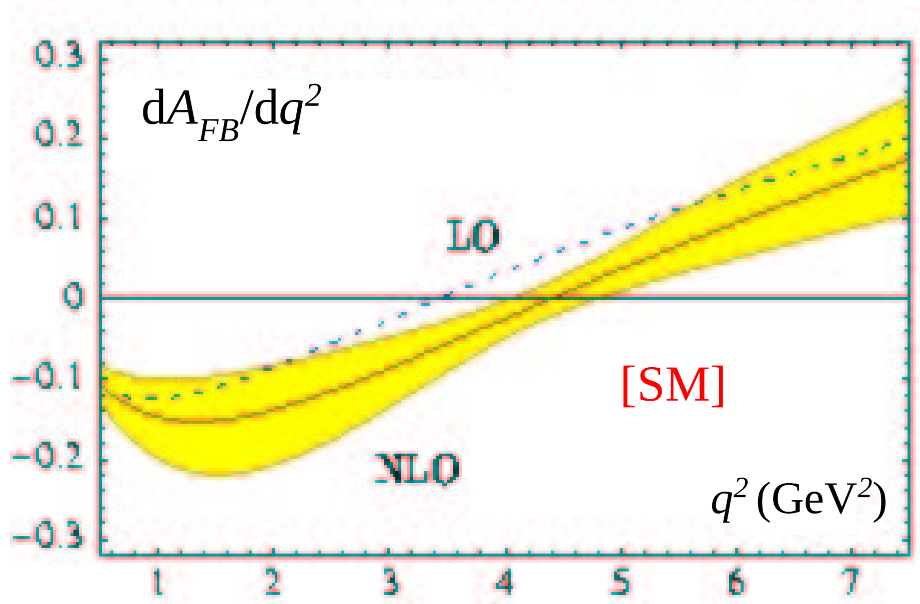

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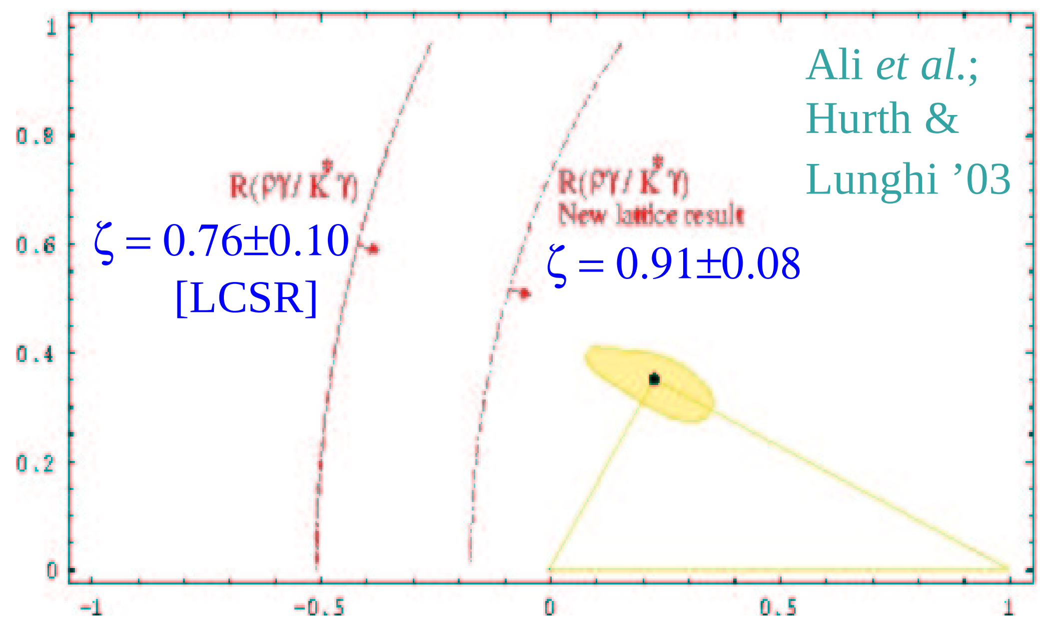
B) $$R(\rho\gamma/K^*\gamma) = \frac{B(B \to \rho\gamma)}{B(B \to K^*\gamma)}$$

$$= \frac{|V_{td}|^2}{|V_{ts}|^2} \frac{(M_B^2 - M_\rho^2)^3}{(M_B^2 - M_K^2)^3} \zeta^2 (1 - \Delta R)$$

f.f. ratio at $q^2=0$ in the HQ limit (→ ζ^2)

$O(\alpha_s)$ & power suppress. terms ($\pm$ 10%) (→ $(1-\Delta R)$)

$R(\rho\gamma/K^*\gamma) < 0.047$ [90%CL, BaBar '03]



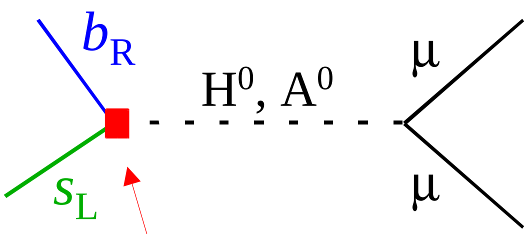
$B_{(s,d)} \to l^+ l^-$

A special case among exclusive *B* decays

- No vector−current contribution [th. error of the s.d. calculation ~ 1% !]
- Hadronic matrix element relatively simple [f_B within the SM]
- Very clean signature
- Strong sensitivity to scalar currents beyond the SM

⇒ order−of−magnitude enhancements possible in multi−Higgs models, even without new flavor structures [SUSY @ large tanβ]

Babu & Kolda, '00
+
wide literature
in the last 3 yrs.



$$A_{scalar} \sim \frac{m_b m_\mu}{M_A^2} \epsilon \tan^3\beta$$

Effective scalar FCNC coupling which *necessarily appears* in SUSY and which is *not suppressed* in the limit of heavy SUSY particles

strong Yukawa suppr. which prevent to observe this effect in allowed transitions such as $B_s \to X\, l^+ l^-$

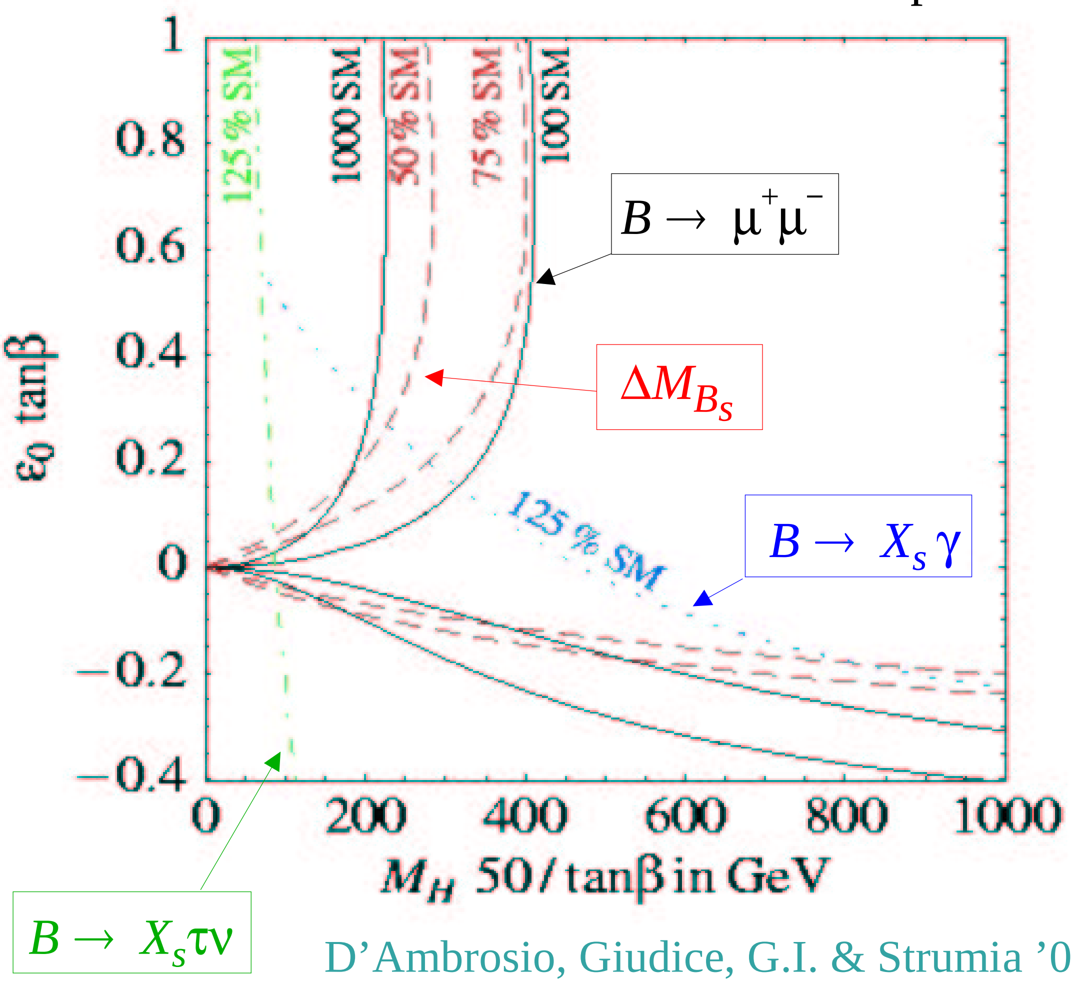
very strong dependence on tanβ [$\epsilon \sim 1/16\pi^2$]

$$B(B_s \to \mu^+\mu^-)^{SM} \approx 3\times10^{-9} \quad < 9.5\times10^{-7} \text{ 90\% CL CDF '03}$$

$$B(B_d \to \mu^+\mu^-)^{SM} \approx 1\times10^{-10} \quad < 1.6\times10^{-7} \text{ 90\% CL BELLE '03}$$

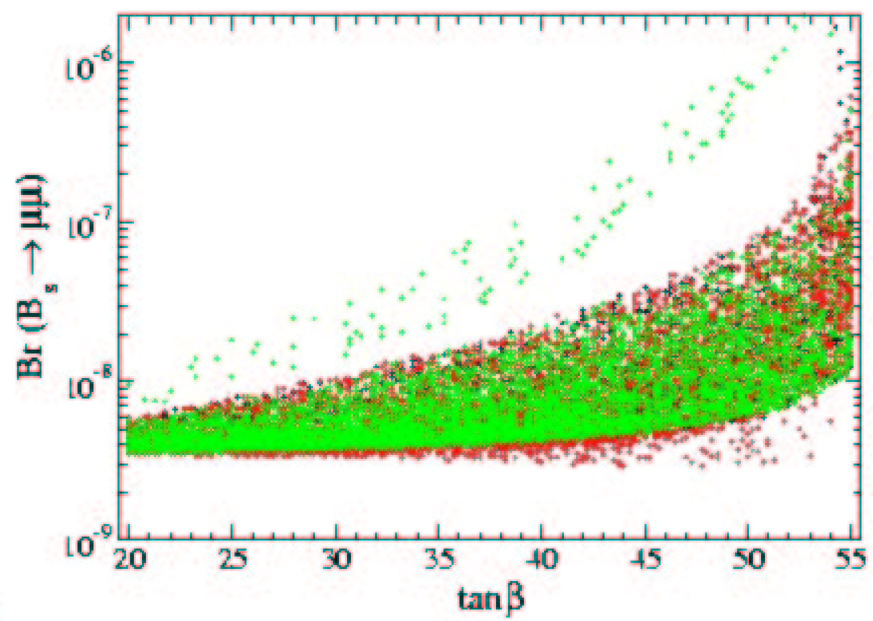
Even the present (weak) bounds put very significant constraints on the SUSY param. space $\Rightarrow$ great discovery potential for future searches at hadronic machines!

General SUSY−2HDM exlusion plot



D'Ambrosio, Giudice, G.I. & Strumia '02

MSUGRA expectations



Kane, Kolda, Lennon '03

Conclusions

The *flavor problem* is one of the most fascinating puzzles in particle physics and rare decays are the key missing pieces which are necessary to reveal the final picture [*the underlying flavor symmetry*]

Experiments at *B* factories have just reached a level of precision which will allow us to extract, in a short time, some of these pieces, *but this is only the beginning...*