Mixing at the B-Factories

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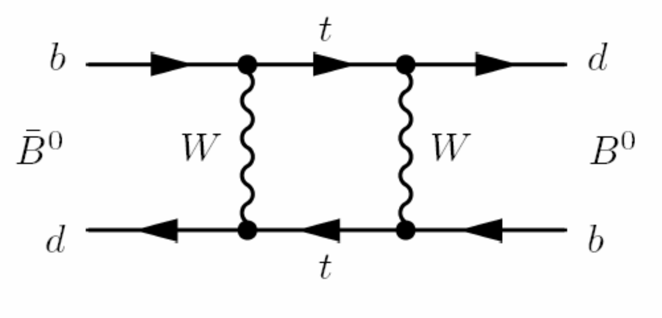
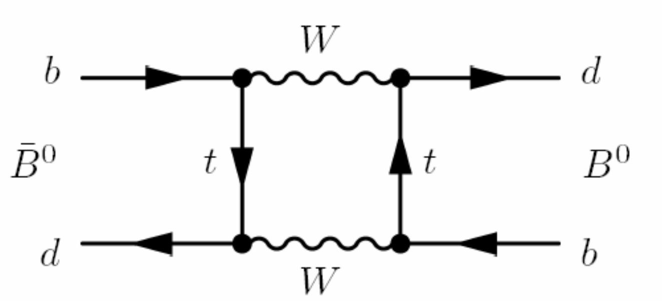
Beauty 2003 10/16/03
the “Iron City”

Neutral B Oscillations





Flavor decomposition:

$$|\psi(t)\rangle = a(t)\left|B^0\right\rangle + b(t)\left|\overline{B}^0\right\rangle$$

Time propagation:

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(\mathbf{M} - \tfrac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

Effective Hamiltonian matrix:

$$M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Standard Assumptions

$$M - \frac{i}{2}\Gamma = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

If CPT is a good symmetry: $M_{11} = M_{22} = \mathrm{M}$ and $\Gamma_{11} = \Gamma_{22} = \Gamma$

Eigenvalues: $\lambda_{H,L} = (M - i\Gamma/2) \pm \frac{q}{p}(M_{12} - i\Gamma_{12}/2)$

$$\Delta m = M_H - M_L = 2|M_{12}|$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|M_{12}|\mathrm{Re}(\Gamma_{12}/M_{12})$$

Expected to be small in the SM and often taken to be ~0

. . . Neutral B Oscillations

Eigenvectors:

$$|B_H\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$

$$|B_L\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

$$\left|\frac{q}{p}\right|^2 \approx 1 - \mathrm{Im}\frac{\Gamma_{12}}{M_{12}}$$

Indirect CP violation:

$$\text{If } \mathrm{P}(\mathrm{B}^0 \to \overline{\mathrm{B}}^0) = \mathrm{P}(\overline{\mathrm{B}}^0 \to \mathrm{B}^0) \text{ then } \left|\frac{q}{p}\right|^2 = 1$$

Dilepton Mixing

At the B-Factories the B^0's are produced in pairs such that

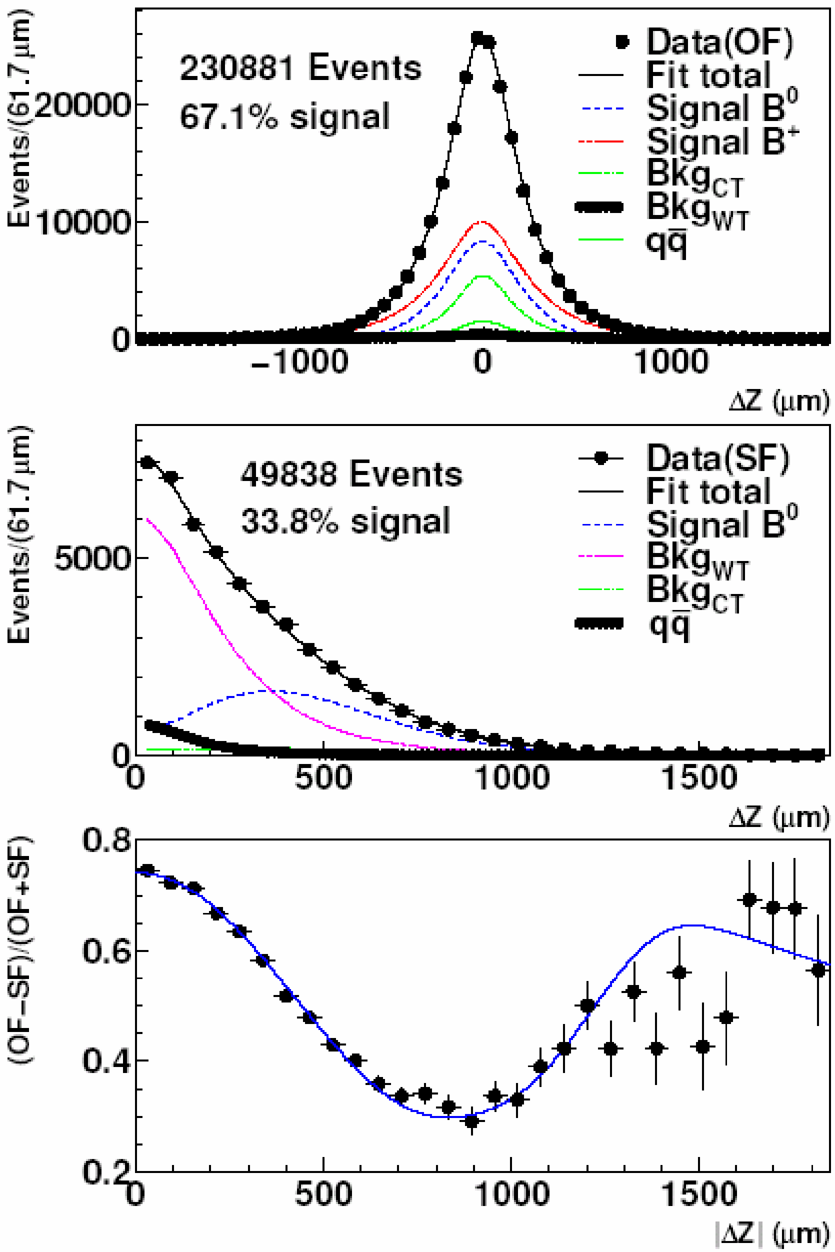
$$\left|\Psi(t=0)\right\rangle = \frac{1}{\sqrt{2}}\left[\left|B^0\right\rangle\left|\overline{B}^0\right\rangle - \left|\overline{B}^0\right\rangle\left|B^0\right\rangle\right]$$

The study of like- and opposite-sign dileptons provides a conceptually and experimentally simple way to observe the mixing, which is sensitive to Δm

Same flavor: $$P(\ell^{\pm}\ell^{\pm},\Delta t) \propto e^{-\Gamma|\Delta t|}\left[1-\cos(\Delta m_d \Delta t)\right]$$

Opposite flavor: $$P(\ell^{+}\ell^{-},\Delta t) \propto e^{-\Gamma|\Delta t|}\left[1+\cos(\Delta m_d \Delta t)\right]$$

. . . Dilepton Mixing



Belle: 29.4 fb^{-1} PRD 67 052004

The relative populations of the same-flavor and opposite-flavor samples as well as the time dependence are important. The measurement is characterized by high statistics.

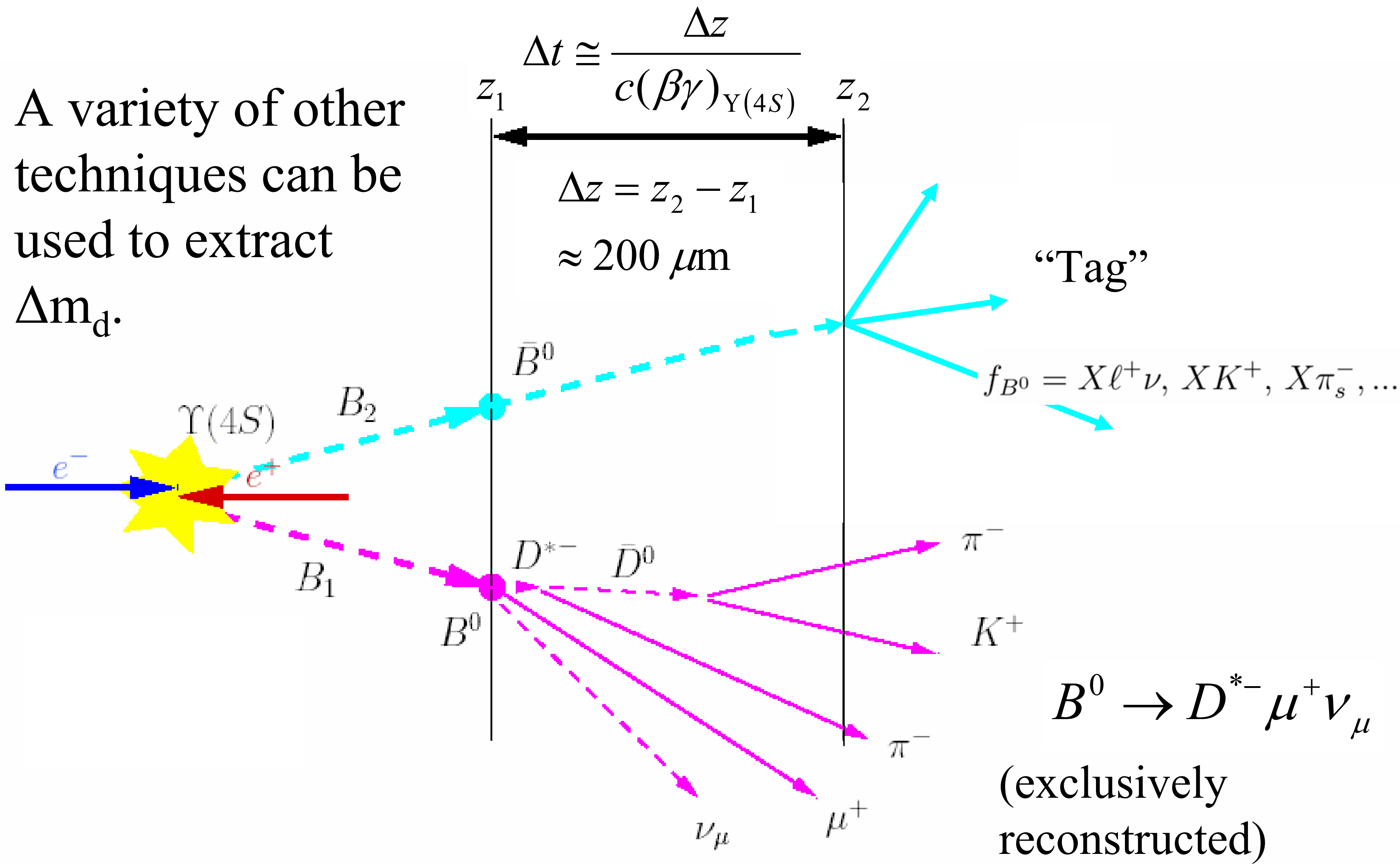
$$\Delta m_d = 0.503 \pm 0.008 \pm 0.010\ \text{ps}^{-1}$$

$$f_+ / f_0 = 1.01 \pm 0.03 \pm 0.09$$

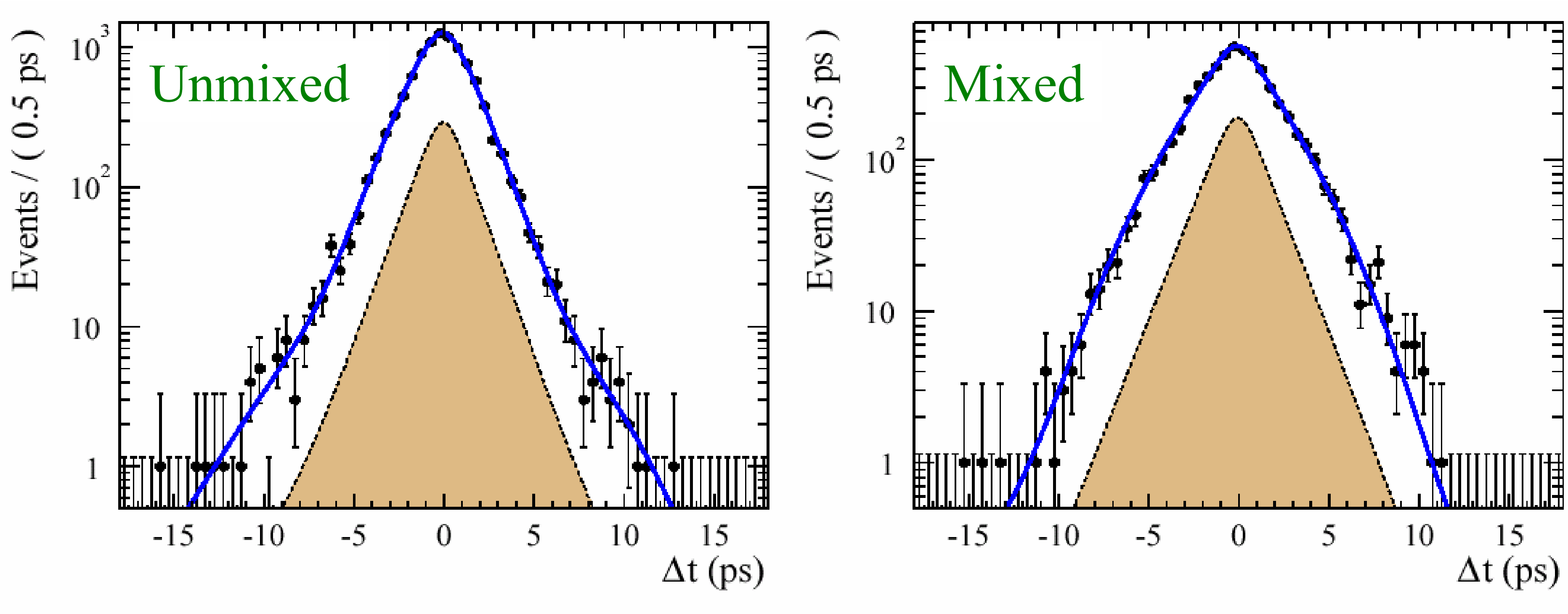
In addition to Δm_d, the measurement yields information on charged to neutral B production.

Other Measurements of Δm_d

A variety of other techniques can be used to extract Δm_d.



Other Measurements of Δm_d

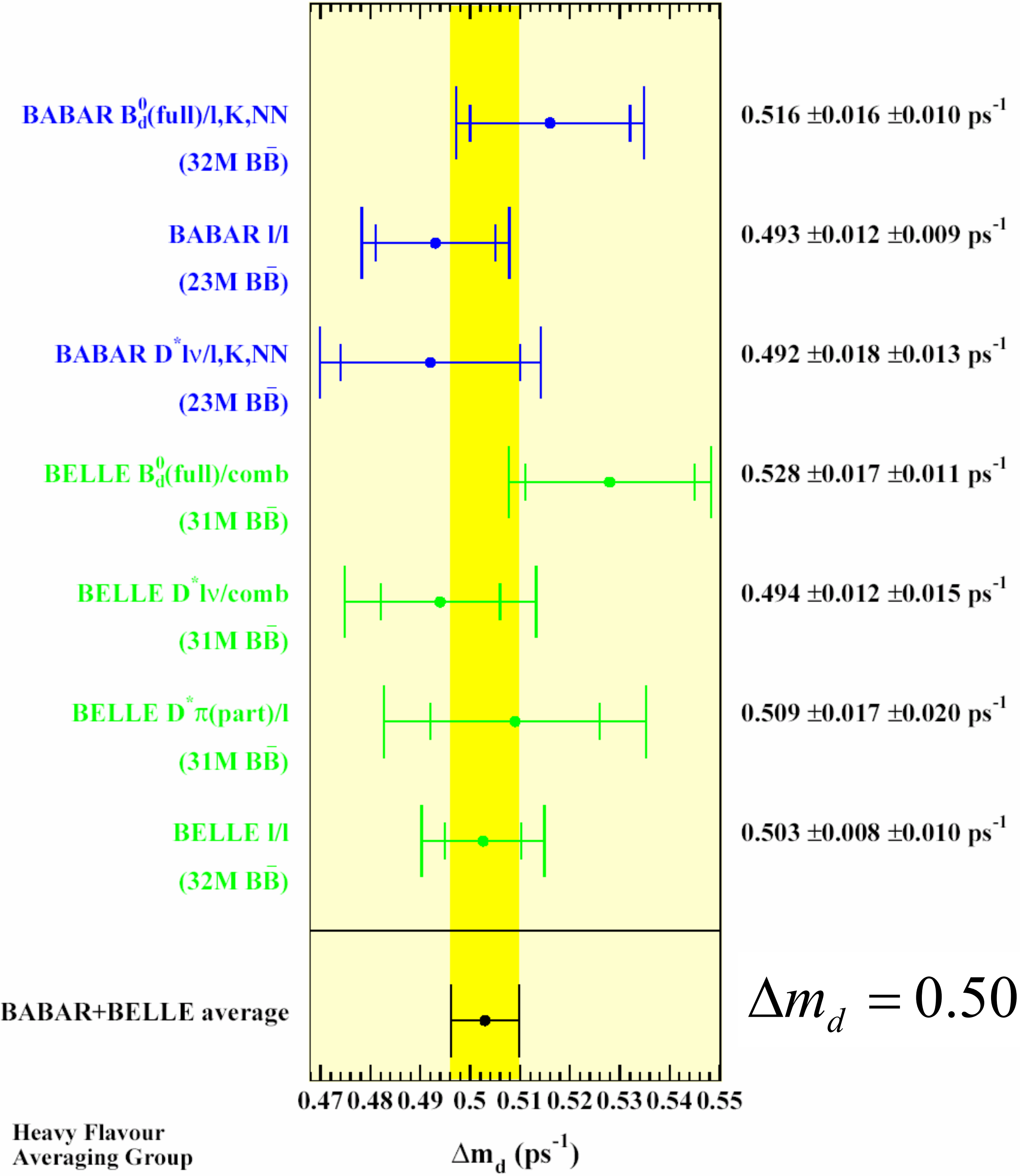


BaBar: 23M *B*-pairs
hep-ex/0212017
submitted to PRD

$$\tau_{B^0} = (1.523^{+0.024}_{-0.023} \pm 0.022)\ \text{ps}$$

$$\Delta m_d = (0.492 \pm 0.018 \pm 0.013)\ \text{ps}^{-1}.$$

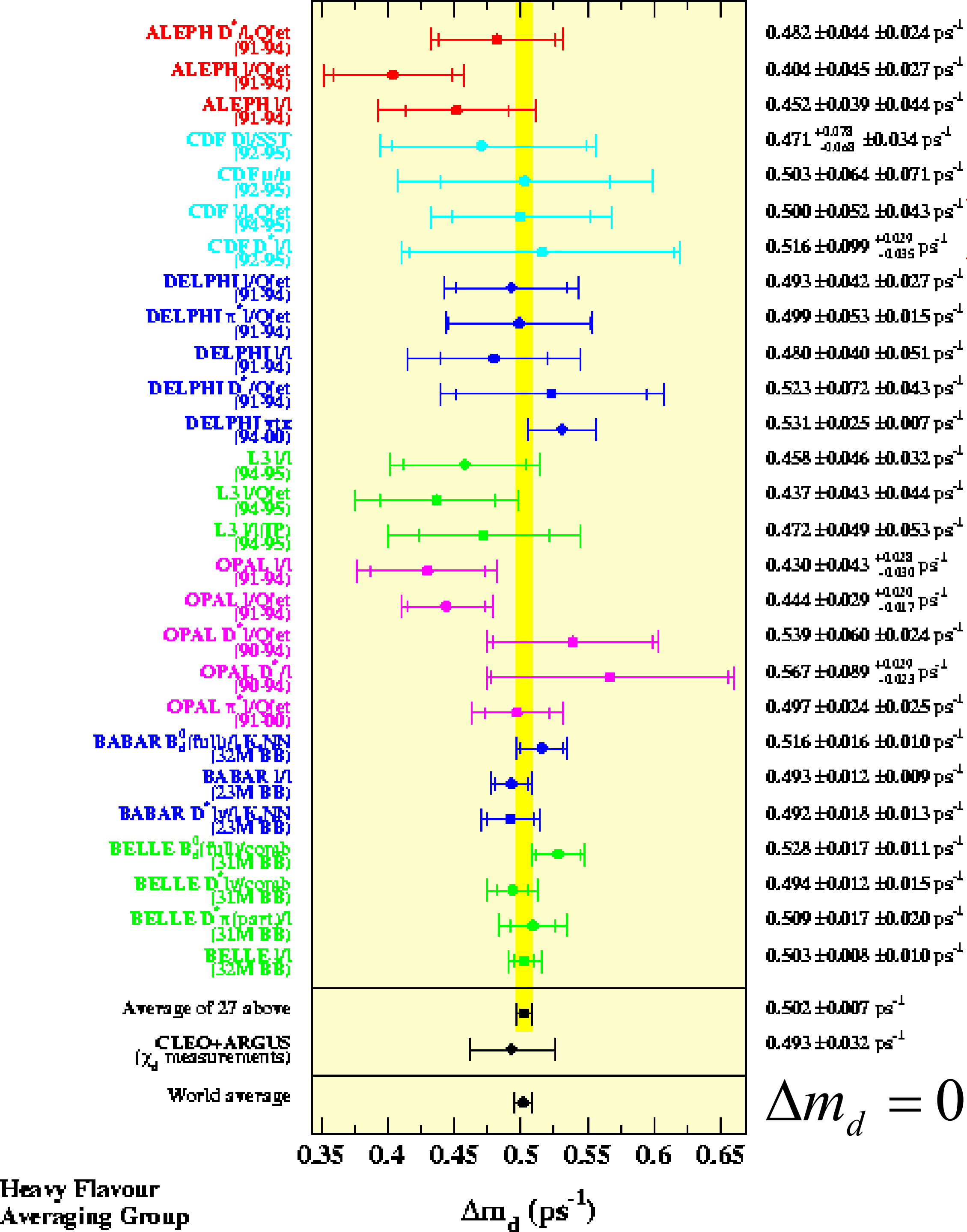
Other Measurements of Δm_d



Belle & BaBar Combined

$$\Delta m_d = 0.507 \pm 0.007 \text{ ps}^{-1}$$

Other Measurements of Δm_d



All experiments combined.

Δm_d is now very well measured!

$$\Delta m_d = 0.502 \pm 0.006 \text{ ps}^{-1}$$

More Dileptons

One call also search for CP violation in the mixing by looking for an asymmetry in the number of like-sign dileptons:

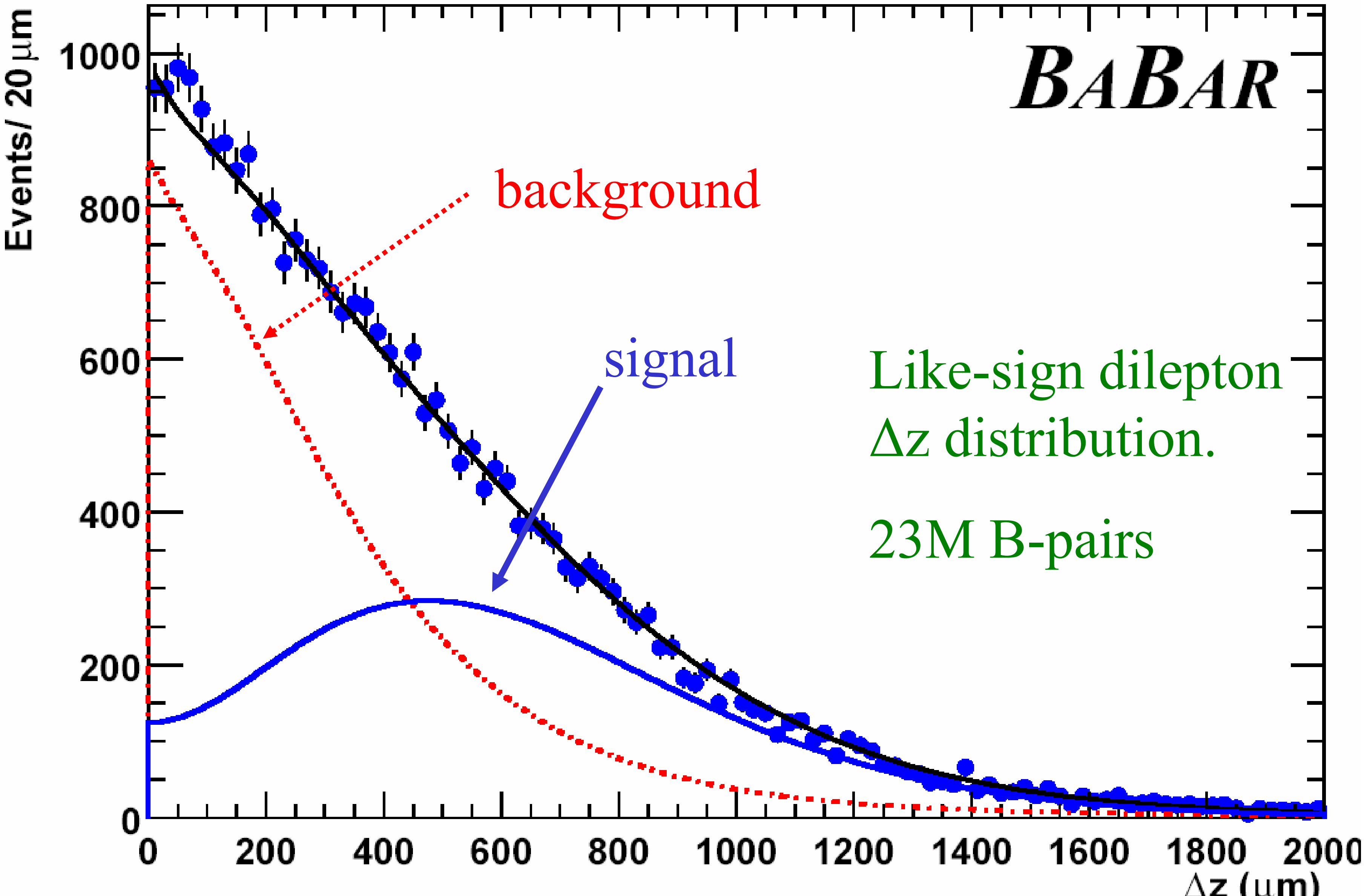
$$A_{T/CP} = \frac{P(\overline{B}^0 \to B^0) - P(B^0 \to \overline{B}^0)}{P(\overline{B}^0 \to B^0) + P(B^0 \to \overline{B}^0)}$$

$$= \frac{1 - |q/p|^4}{1 + |q/p|^4}.$$

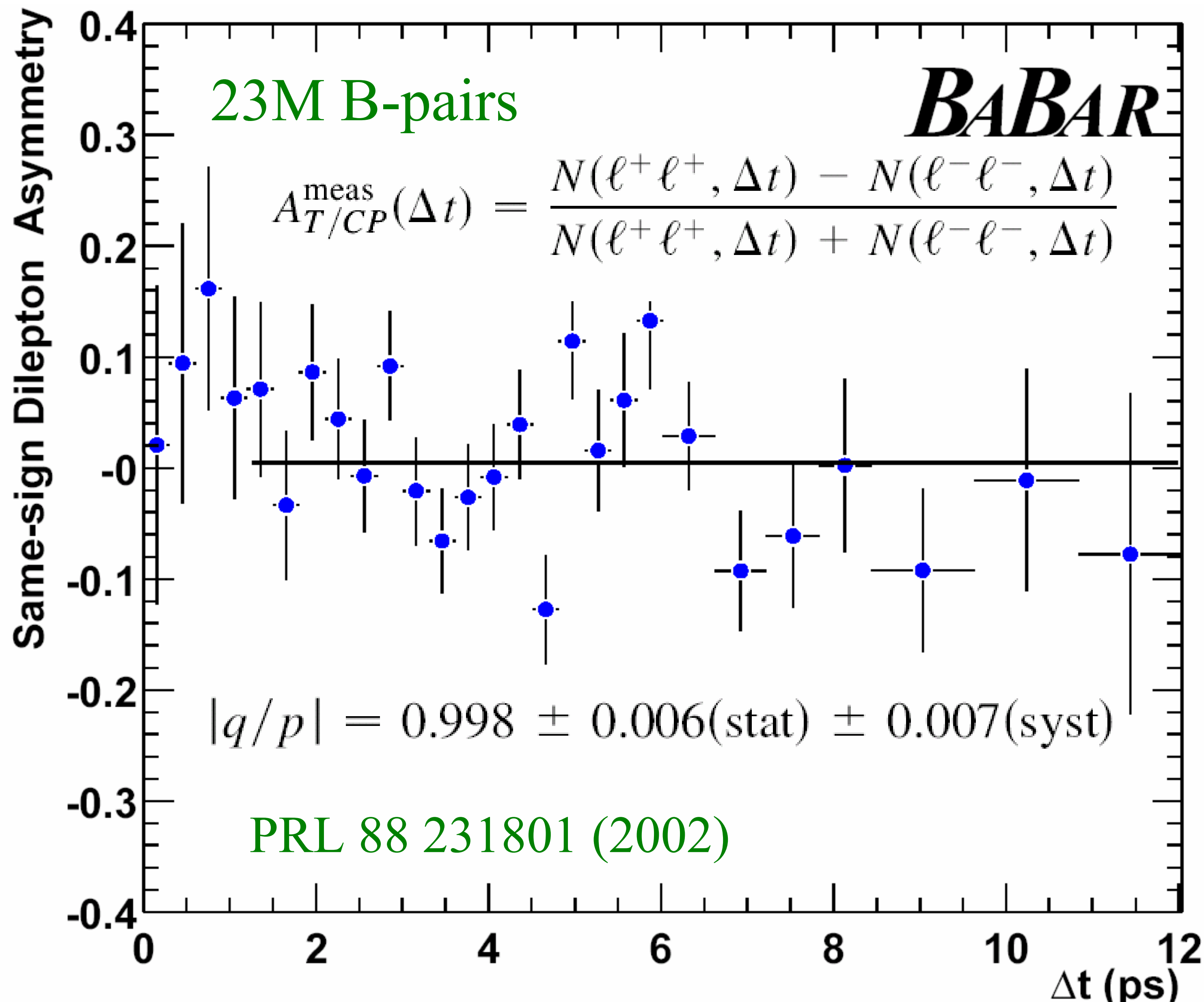
$$\left|\frac{q}{p}\right|^2 = 1 - \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \qquad \left|\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right| \propto \Gamma_{12}\frac{m_c^2}{m_b^2} < 10^{-3}$$

In SM effects are expected to be small.

CP & T Violation in Dileptons



CP & T Violation in Dileptons



Thinking the unthinkable: CPT Violation

The mixing analysis can be extended to incorporate possible CPT violating effects via the inclusion of the parameter θ. If CPT is a good symmetry $\theta=\pi/2$.

$$P^{\text{unm}}: \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[(1-|\cos\theta|^2)\cos(\Delta m_d \Delta t)$$

$$+1+|\cos\theta|^2 - 2\Im(\cos\theta)\sin(\Delta m_d \Delta t)]$$

$$P^{\text{mix}}: \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}|\sin\theta|^2[1-\cos(\Delta m_d \Delta t)].$$

Note that for $\theta = \frac{\pi}{2} + \varepsilon \quad \sin\theta \cong 1 - \frac{\varepsilon^2}{2}$ and $\cos\theta \cong -\varepsilon$

Thinking the unthinkable: CPT Violation

Belle did a second fit to their dilepton data, allowing for the possibility of CPT violation.

$$P^{unm}_{\ell^+\ell^-} \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[(1-|\cos\theta|^2)\cos(\Delta m_d \Delta t)$$

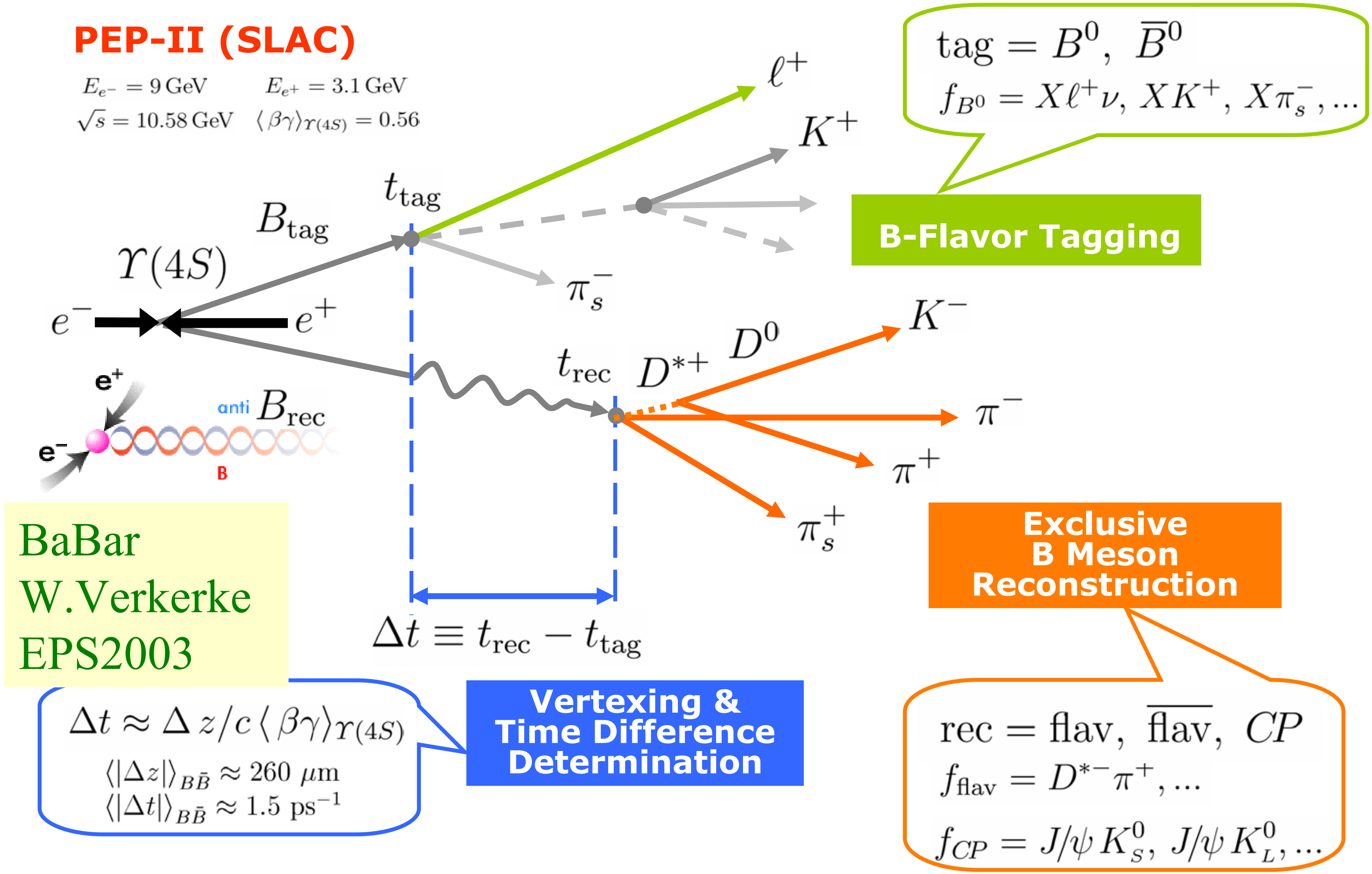
$$+1+|\cos\theta|^2 - 2\Im(\cos\theta)\sin(\Delta m_d \Delta t)]$$

$$\Re(\cos\theta) = 0.00 \pm 0.12(\text{stat}) \pm 0.01(\text{syst}),$$

$$\Im(\cos\theta) = 0.03 \pm 0.01(\text{stat}) \pm 0.03(\text{syst}).$$

Belle: 29.4 fb^{-1} N.Hastings et al.PRD 67 052004

Coherent Time Evolution at the Υ(4S)



Time dependent B decay rates allowing ΔΓ≠0, CP/T/CPT violation

BaBar has introduced a new parameterization wherein a fit is done to samples of the form

$$e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0 \to \begin{Bmatrix} f_{\text{flavor}}, f_{\text{tag}} \\ f_{CP}\ \ , f_{\text{tag}} \end{Bmatrix}$$

Fit to

$$p\left(\Delta t;\ \text{sgn}(\text{Re}\,\lambda_{CP})\frac{\Delta\Gamma}{\Gamma}, \left|\frac{q}{p}\right|, \left(\frac{\text{Re}\,\lambda_{CP}}{|\lambda_{CP}|}\right)\text{Re}\,z, \text{Im}\,z\right)$$

where $\lambda_{CP} = \frac{\overline{A}_{CP}}{A_{CP}}\frac{q}{p}$ and $\Delta\Gamma = \Gamma_H - \Gamma_L$

BaBar
W.Verkerke
EPS2003
hep-ex/0303043

$$z = \frac{(M_{11}-M_{22}) - \frac{i}{2}(\Gamma_{11}-\Gamma_{22})}{4\left(\Delta m - \frac{i}{2}\Delta\Gamma\right)} \cong \cot\theta$$

$z \neq 0 \Rightarrow$ ~~CPT~~

Time dependent B decay rates allowing ΔΓ≠0, CP/T/CPT violation: fit results

$$\mathrm{sgn}(\mathrm{Re}\,\lambda_{CP})\frac{\Delta\Gamma}{\Gamma} = -0.008 \pm 0.037 \pm 0.018$$

$$\left|\frac{q}{p}\right| = 1.029 \pm 0.013 \pm 0.011$$

$$\left(\frac{\mathrm{Re}\,\lambda_{CP}}{|\lambda_{CP}|}\right)\mathrm{Re}\,z = 0.014 \pm 0.035 \pm 0.034$$

$$\mathrm{Im}\,z = 0.038 \pm 0.029 \pm 0.025$$

BaBar 82 fb^{-1}

hep-ex/0303043

The Future: Δm_d

Current world average (HFAG): $\Delta m_d = 0.502 \pm 0.006$

	Single Measurement Errors		
	Method(s)	**Statistical**	**Systematic***
Now (30 fb^{-1})	Dilepton	±0.008	±0.010
	"Hadronic"	±0.016	±0.010
Future (300 fb^{-1})	Dilepton	±0.003	±0.004 ~ ±0.007
	"Hadronic"	±0.005	±0.005 ~ ±0.007

We are nearing the end of the road, but a ~twofold improvement may still be in the cards.

*Systematic error projections from C.Bozzi CKM Workshop CERN 2002

The Future: CPT Parameters

BaBar (82 fb^{-1})	$\mathrm{Im}(z) = 0.038 \pm 0.029 \pm 0.025$
	$\mathrm{Re}(z) = 0.014 \pm 0.035 \pm 0.034$
Belle (29 fb^{-1})	$\mathrm{Im}(\cos\theta) = 0.03 \pm 0.01 \pm 0.03$
	$\mathrm{Re}(\cos\theta) = 0.00 \pm 0.12 \pm 0.02$

Once again, there is room for some improvement, but we are bumping up against systematic errors.

We shouldn't forget the possibility of new ideas!