Ultra-Rare B Decays

Benjamín Grinstein

bgrinstein@ucsd.edu

UCSD

Decay	SM Prediction
$B_u^+ \to e^+\nu_e$	6.9×10^{-12}
$B_u^+ \to \mu^+\nu_\mu$	9.9×10^{-7}
$B_u^+ \to \tau^+\nu_\tau$	6.6×10^{-5}
$B_d^0 \to e^+e^-$	6.9×10^{-12}
$B_d^0 \to \mu^+\mu^-$	1.1×10^{-10}
$B_d^0 \to \tau^+\tau^-$	3.1×10^{-8}
$B_u^+ \to \ell^+\nu\gamma$	$10^{-7} - 10^{-5}$
$B_s^0 \to \ell^+\ell^-\gamma(e,\mu)$	$(2-5) \times 10^{-9}$
$B_d^0 \to \ell^+\ell^-\gamma$	$(3-6) \times 10^{-10}$
$B_s^0 \to \gamma\gamma$	—
$B_d^0 \to X_s\nu\bar{\nu}$	$(4.1 \pm 1.0) \times 10^{-5}$
$B_d^0 \to K\sum_i \nu_i\bar{\nu}_i$	$(2-9) \times 10^{-6}$
$B_d^0 \to K^*\sum_i \nu_i\bar{\nu}_i$	$(0.2-2) \times 10^{-5}$

BaBar-Book (Ultra?) Rare Decays

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$B_u^+ \to e^+\nu_e$	6.9×10^{-12}
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$B_u^+ \to \ell^+\nu\gamma$	$10^{-7} - 10^{-5}$
$B_s^0 \to \ell^+\ell^-\gamma(e,\mu)$	$(2-5) \times 10^{-9}$
$B_d^0 \to \ell^+\ell^-\gamma$	$(3-6) \times 10^{-10}$
$B_s^0 \to \gamma\gamma$	—
$B_d^0 \to X_s\nu\bar{\nu}$	$(4.1 \pm 1.0) \times 10^{-5}$
$B_d^0 \to K\sum_i \nu_i\bar{\nu}_i$	$(2-9) \times 10^{-6}$
$B_d^0 \to K^*\sum_i \nu_i\bar{\nu}_i$	$(0.2-2) \times 10^{-5}$

Decay	SM Prediction
$B \to X_s\gamma$	N.U.
$B \to V\gamma$	N.U.
$B \to X_s\ell^+\ell^-$	N.U.
$B \to (P,V)\ell^+\ell^-$	N.U.

$B^{\pm} \to \ell^{\pm}\nu$

S. W. Baek and Y. G. Kim, PRD **60**, 077701 (1999).
H. K. Dreiner, G. Polesello and M. Thormeier, PRD **65**, 115006 (2002).
A. G. Akeroyd and S. Recksiegel, Phys. Lett. B **541**, 121 (2002).
A. G. Akeroyd and S. Recksiegel, Phys. Lett. B **554** (2003) 38

$$\Gamma(B_q^+ \to \ell^+\nu_\ell) = \frac{G_F^2 m_{B_q} m_l^2 f_{B_q}^2}{8\pi} |V_{qb}|^2 \left(1 - \frac{m_l^2}{m_{B_q}^2}\right)^2$$

- SM's $V - A$ vertex $\Rightarrow$ Helicity suppression
- not supressed in extensions with other tensor structure
  - $B \to X_q e\nu$ is SM dominated (new physics may hide)
  - $\bar{b}qe\nu$ coupling slightly constrained
  - $B^{\pm} \to e^{\pm}\nu$ could show huge deviations

SM predictions and experimental bounds

Decay	SM Prediction	CLEO	BELLE	LEP / Tevatron
$B_u^+ \to e^+\nu_e$	9.2×10^{-12}	$\leq 1.5 \times 10^{-5}$	$\leq 4.7 \times 10^{-6}$	$\times$
$B_u^+ \to \mu^+\nu_\mu$	3.9×10^{-7}	$\leq 2.1 \times 10^{-5}$	$\leq 6.5 \times 10^{-6}$	$\times$
$B_u^+ \to \tau^+\nu_\tau$	8.7×10^{-5}	$\leq 8.4 \times 10^{-4}$	$\times$	$\leq 5.7 \times 10^{-4}$
$B_c^+ \to e^+\nu_e$	2.5×10^{-9}	$\times$	$\times$	$\times$
$B_c^+ \to \mu^+\nu_\mu$	1.1×10^{-4}	$\times$	$\times$	$\times$
$B_c^+ \to \tau^+\nu_\tau$	2.6×10^{-2}	$\times$	$\times$	$\times$

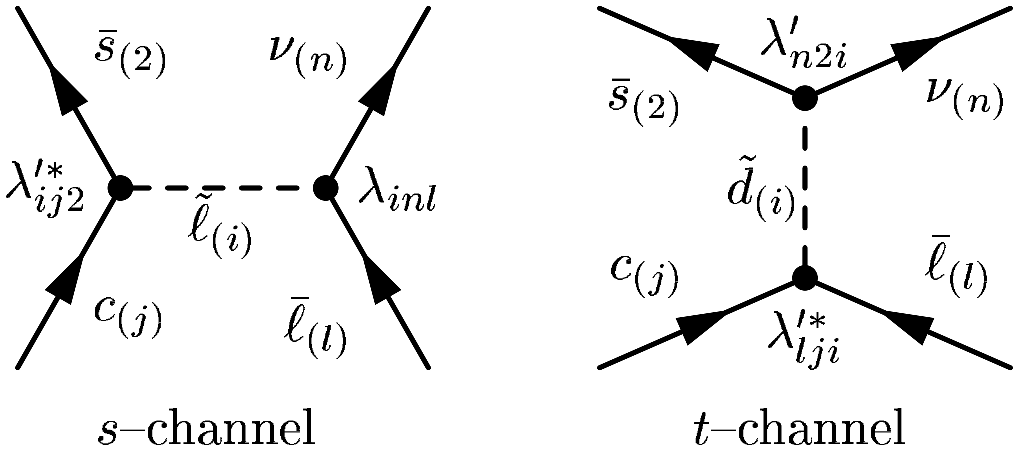
CLEO: M. Artuso *et al.* [CLEO Collaboration], Phys. Rev. Lett. **75**, 785 (1995).
T. E. Browder *et al.* [CLEO Collaboration], Int. J. Mod. Phys. A **16S1B**, 636 (2001).
BELLE–CONF–0127, http://belle.kek.jp/conferences/LP01-EPS/.
L3: M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **396**, 327 (1997).

R Parity Violation

R-breaking Superpotential:

$$W = \frac{1}{2}\lambda_{ijk}L_iL_jE^c_k + \lambda'_{ijk}L_iQ_jD^c_k + \frac{1}{2}\lambda''_{ijk}U^c_iD^c_jD^c_k$$

Exclude λ''_{ijk} (proton stability).



squark/slepton exchange

Sample upper bounds on products of couplings from $B \to l\nu$.

Decay Mode	Combinations Constrained	Upper bound
$B^- \to e^- \bar{\nu}$	$\lambda_{131}\lambda'_{113}$	7.3×10^{-5}
	$\lambda_{131}\lambda'_{123}$	3.2×10^{-4}
	$\lambda_{131}\lambda'_{323}$	3.2×10^{-4}
	$\lambda_{231}\lambda'_{213}$	7.3×10^{-5}
	$\lambda_{231}\lambda'_{223}$	3.2×10^{-4}
	$\lambda_{231}\lambda'_{233}$	2.0×10^{-2}
	$\lambda_{231}\lambda'_{323}$	3.2×10^{-4}
$B^- \to \mu^- \bar{\nu}$	$\lambda_{132}\lambda'_{113}$	8.7×10^{-5}
	$\lambda_{132}\lambda'_{123}$	3.8×10^{-4}
	$\lambda_{132}\lambda'_{323}$	3.8×10^{-4}
	$\lambda_{232}\lambda'_{213}$	8.7×10^{-5}
	$\lambda_{232}\lambda'_{223}$	3.8×10^{-4}
$B^- \to \tau^- \bar{\nu}$	$\lambda_{123}\lambda'_{113}$	5.1×10^{-4}
	$\lambda_{233}\lambda'_{213}$	5.1×10^{-4}
	$\lambda_{233}\lambda'_{313}$	5.1×10^{-4}

$B \to \gamma \ell^+ \ell^-$

- Historical mess with form factors
  - Ward Identities (gauge invariance) not obeyed by model calculations
  - New models of form factors impose Ward identities
  - Same issues in $B \to V\gamma$, ($V = K^*, \ldots$)
- Charge asymmetries
- General New Phsyics Analysis

Ward Identities in FF's for $B \to \gamma\ell^+\ell^-$

C. Q. Geng, C. C. Lih and W. M. Zhang, PRD**62**,074017(2000)
B. Grinstein and D. Pirjol, PRD**62**,093002(2000)
F. Kruger and D. Melikhov, PRD**67**,034002(2003)

$$\langle\gamma(q,\epsilon) f|\mathcal{O}(0)|B(v)\rangle = -ie\epsilon^*_\mu \int \mathrm{d}^4x e^{iq\cdot x}\langle f|\mathrm{T} j^{\mathrm{e.m.}}_\mu(x)\,\mathcal{O}(0)|B(v)\rangle$$

EM current: $j^{\mathrm{e.m.}}_\mu = +\frac{2}{3}(\bar u\gamma_\mu u + \bar c\gamma_\mu c) - \frac{1}{3}(\bar d\gamma_\mu d + \bar s\gamma_\mu s + \bar b\gamma_\mu b)$.

$$-iq_\mu \int \mathrm{d}^4x e^{iq\cdot x}\langle f|\mathrm{T} j^{\mathrm{e.m.}}_\mu(x)\,\mathcal{O}(0)|B(v)\rangle = \int \mathrm{d}^3x e^{-i\vec q\cdot\vec x}\langle f|[j^{\mathrm{e.m.}}_0(\vec x)\,,\mathcal{O}(\vec 0)]|B(v)\rangle$$

$[j^{\mathrm{e.m.}}_0(\vec x)\,,\mathcal{O}(\vec 0)] \neq 0 \Leftrightarrow \mathcal{O}$ has electric charge as in, eg,

$B^+ \to \gamma e^+\nu$, $B \to \pi(\rho)\gamma e^+\nu$ or $\bar B \to D^{(*)}\gamma e^+\nu$.

(for $|f\rangle = |0\rangle$, $|\pi\rangle(|\rho\rangle)$, $|D^{(*)}\rangle$, respectively)

Example: axial current

Take $\mathcal{O} = \bar{b}\gamma_\nu\gamma_5 q$ and $|f\rangle = |0\rangle \Rightarrow$

$$-iq_\mu \int \mathrm{d}^4x e^{iq\cdot x}\langle 0|\mathrm{T}j_\mu^{\mathrm{e.m.}}(x)\,(\bar{b}\gamma_\nu\gamma_5 q)(0)|B(v)\rangle = (Q_b - Q_q)\langle 0|\bar{b}\gamma_\nu\gamma_5 q|B(v)\rangle = (Q_b - Q_q) f_B m_B v_\mu$$

Parametrize LHS with five form-factors $f_i(q^2, v\cdot q)$

$$-i\int \mathrm{d}^4x e^{iq\cdot x}\langle 0|\mathrm{T}j_\mu^{\mathrm{e.m.}}(x)\,(\bar{b}\gamma_\nu\gamma_5 q)(0)|B(v)\rangle = f_1 g_{\mu\nu} + f_2 v_\mu v_\nu + f_3 q_\mu q_\nu + f_4 q_\mu v_\nu + f_5 v_\mu q_\nu\,.$$

Ward identity implies

$$(v\cdot q) f_2 + q^2 f_4 = (Q_b - Q_q) f_B m_B\,, \qquad f_1 + q^2 f_3 + (v\cdot q) f_5 = 0$$

$$\langle\gamma(q,\epsilon) f|\bar{b}\gamma_\mu\gamma_5 q|B(v)\rangle = -f_5[(v\cdot q)\epsilon_\mu^* - (v\cdot\epsilon^*)q_\mu] + (v\cdot\epsilon^*)v_\mu \frac{1}{v\cdot q}(Q_b - Q_q) f_B m_B$$

Form Factors in $B \to \gamma \ell^+ \ell^-$

$$\begin{aligned}
\langle \gamma(k) | \bar{q} \gamma_\mu \gamma_5 b | \bar{B}_q(p) \rangle &= i e \varepsilon^{*\alpha}(k) [g_{\mu\alpha}(p \cdot k) - p_\alpha k_\mu] \frac{F_A}{M_{B_q}}, \\
\langle \gamma(k) | \bar{q} \gamma_\mu b | \bar{B}_q(p) \rangle &= e \varepsilon^{*\alpha}(k) \epsilon_{\mu\alpha\rho\sigma} p^\rho k^\sigma \frac{F_V}{M_{B_q}} \\
\langle \gamma(k) | \bar{q} \sigma_{\mu\nu} \gamma_5 b | \bar{B}_q(p) \rangle (p-k)^\nu &= e \varepsilon^{*\alpha}(k) [g_{\mu\alpha}(p \cdot k) - p_\alpha k_\mu] F_{TA} \\
\langle \gamma(k) | \bar{q} \sigma_{\mu\nu} b | \bar{B}_q(p) \rangle (p-k)^\nu &= i e \varepsilon^{*\alpha}(k) \epsilon_{\mu\alpha\rho\sigma} p^\rho k^\sigma F_{TV}
\end{aligned}$$

Form Factors in LEET

LEET:
BG& M. Dugan,
J. Charles, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, PRD **60**, 014001 (1999).

$$F_V \simeq F_A \simeq F_{TA} \simeq F_{TV} \simeq \zeta_\perp^\gamma(E, M_B),$$

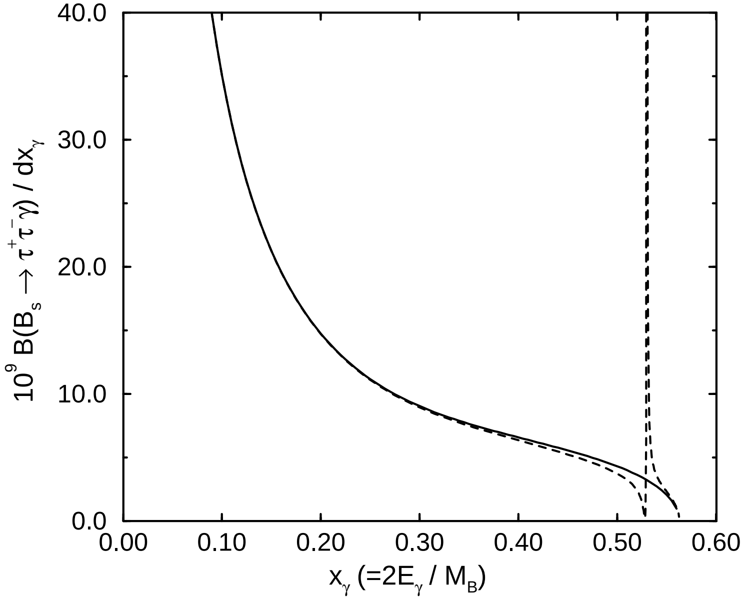
a universal large energy function, with

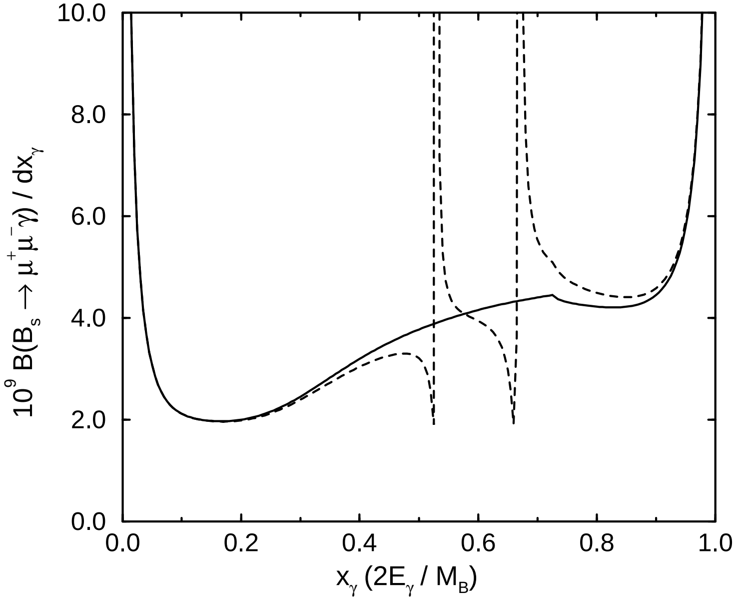
$$\zeta_\perp^\gamma(E, M_B) \propto \frac{f_B M_B}{E}.$$

Coefficient fixed by

$$R \equiv \int dx \, \frac{\phi_B(x)}{x}$$

Light-Front Model



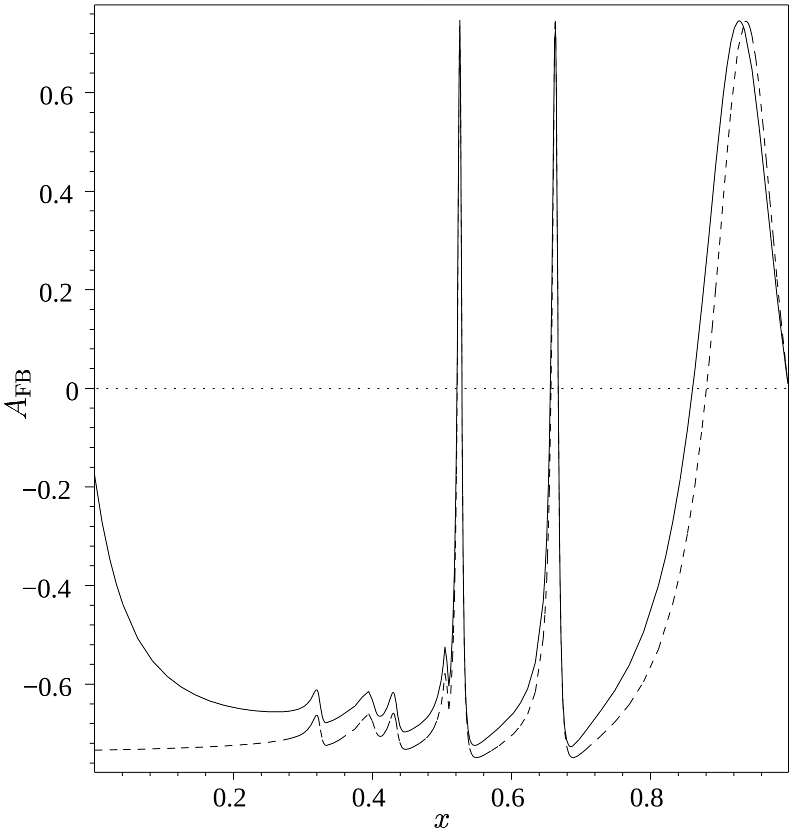


$x_\gamma = 2E_\gamma/M_B$ (Geng et al)

Br$(B_s \to \mu^+\mu^-\gamma) = 8.3 \times 10^{-9}$ $(\delta = 0.01)$

Br$(B_s \to \tau^+\tau^-\gamma) = 1.6 \times 10^{-8}$

LEET Inspired Model-FB asymmetry



- $\bar{B}_s \to \mu^+\mu^-\gamma$
- μ^- FB asymmetry
- solid: LEET inspired
- dashed: LO LEET

(Kruger and Melikhov)

Is zero model dependent?

oscillations? taging? . . .

New Physics

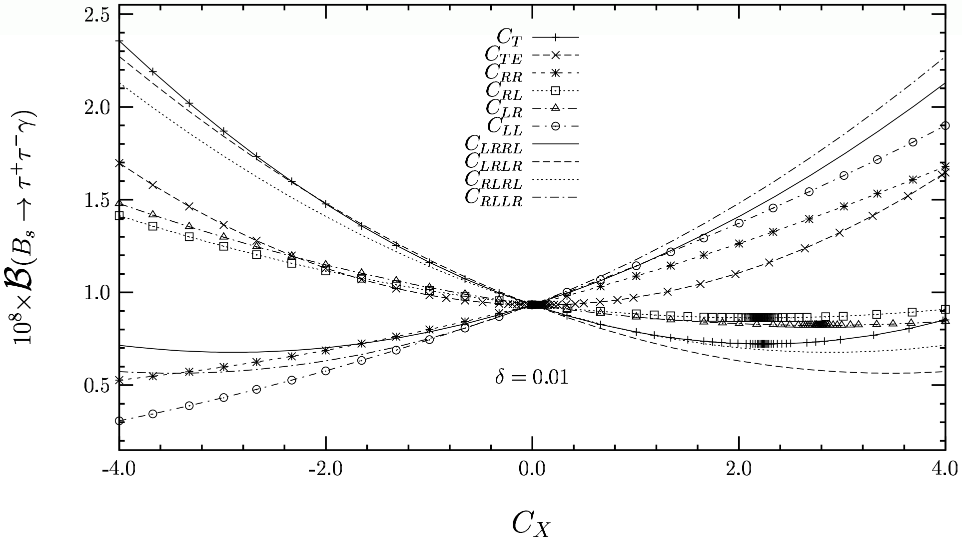
T. M. Aliev, A. Ozpineci and M. Savci, PLB**520**,69(2001)

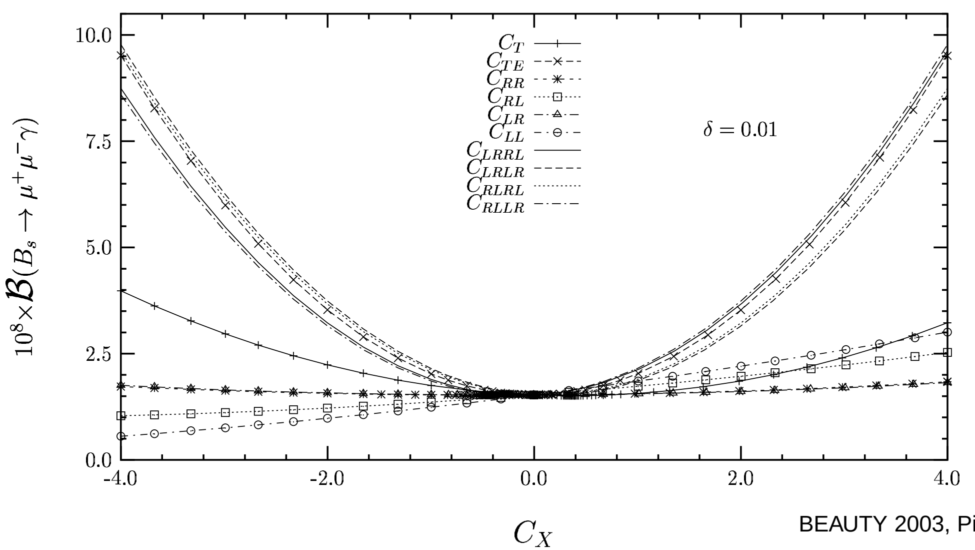
$$
\begin{aligned}
\mathcal{H}_{eff} \;=\; & \frac{G\alpha}{\sqrt{2}\pi} V_{tq} V_{tb}^{*} \Bigg\{ C_{SL}\, \bar{q}_R i\sigma_{\mu\nu} \frac{q^\nu}{q^2} b_L\, \bar{\ell}\gamma^\mu \ell + C_{BR}\, \bar{q}_L i\sigma_{\mu\nu} \frac{q^\nu}{q^2} b_R\, \bar{\ell}\gamma^\mu \ell \\
& + C_{LL}^{tot}\, \bar{q}_L \gamma_\mu b_L\, \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}^{tot}\, \bar{q}_L \gamma_\mu b_L\, \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}\, \bar{q}_R \gamma_\mu b_R\, \bar{\ell}_L \gamma^\mu \ell_L \\
& + C_{RR}\, \bar{q}_R \gamma_\mu b_R\, \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR}\, \bar{q}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{q}_R b_L\, \bar{\ell}_L \ell_R \\
& + C_{LRRL}\, \bar{q}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{q}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{q}\sigma_{\mu\nu} b\, \bar{\ell}\sigma^{\mu\nu} \ell \\
& + i C_{TE}\, \epsilon^{\mu\nu\alpha\beta} \bar{q}\sigma_{\mu\nu} b\, \bar{\ell}\sigma_{\alpha\beta} \ell \Bigg\}
\end{aligned}
$$

In SM

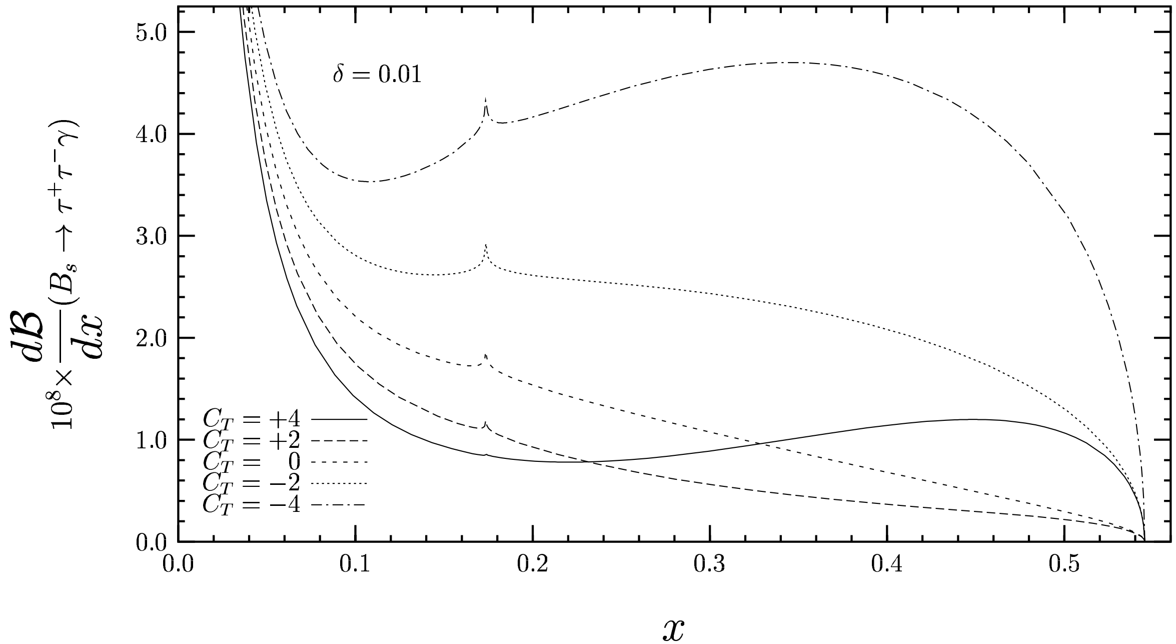
$$
\begin{aligned}
C_{BR} &= -2 m_b C_7^{eff} \\
C_{LL}^{tot} &= C_9^{eff} - C_{10} \\
C_{LR}^{tot} &= C_9^{eff} + C_{10}
\end{aligned}
$$

New Physics effect on Br





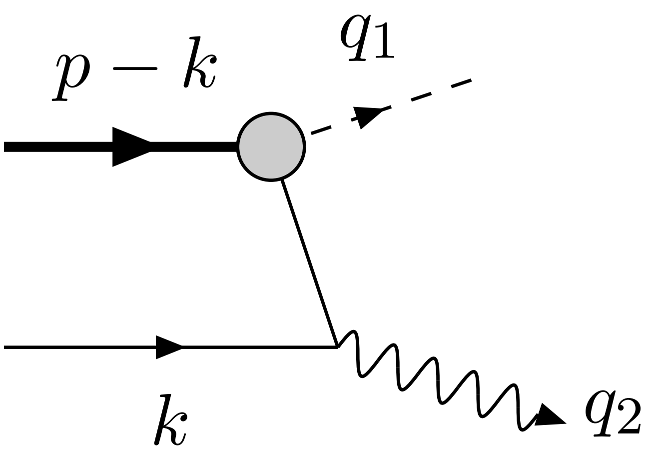
New Physics (C_T) effect on Decay Spectrum



NOTE: FFs from Light Cone QCD sum rules, double poles with two parameters

$B \to \gamma \ell \nu_\ell$, $B \to \gamma\gamma$ **and** $B \to \gamma \ell^+ \ell^-$

From QCD point of view, these are all “the same”



$B \to \gamma \ell \nu_\ell$, $B \to \gamma\gamma$ **and** $B \to \gamma \ell^+ \ell^-$

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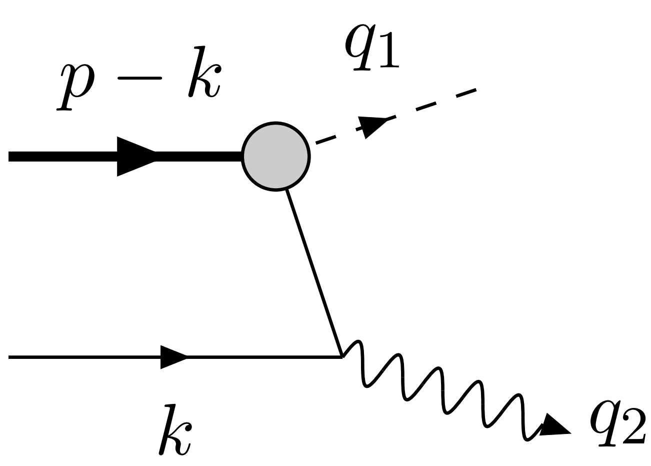


Factorization, SCET, and $B \to \gamma \ell \nu_\ell$, $B \to \gamma\gamma$ and $B \to \gamma \ell^+ \ell^-$

G. P. Korchemsky, D. Pirjol and T. M. Yan, PRD **61**, 114510 (2000)
H. N. Li, PRD **64**, 014019 (2001)
E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649**, 349 (2003)
S. Descotes-Genon and C. T. Sachrajda, NPB **650**, 356 (2003); PLB **557**, 213 (2003)
S. W. Bosch, arXiv:hep-ph/0308319

$$\Gamma(\bar{B}_q \to \gamma\gamma) = \frac{\alpha^2 G_F^2 M_B^5 f_B^2}{144\pi^3} \left(C_7^{\rm eff}\right)^2 |\lambda_t^{(q)}|^2 (C_9^{SCET})^2 \frac{1}{\Lambda_B^2(M_B/2)}$$

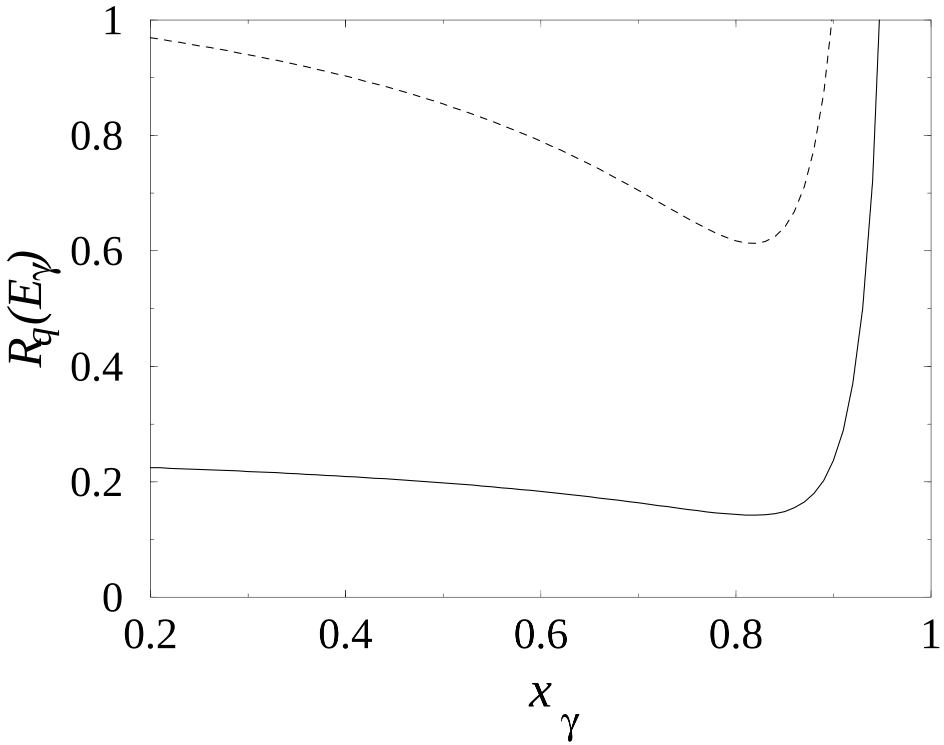
$$\frac{d\Gamma(\bar{B}_q \to \gamma e^+ e^-)}{dE_\gamma} = \frac{\alpha^3 G_F^2 M_B^4 f_B^2}{1728\pi^4} |\lambda_t^{(q)}|^2 \frac{x_\gamma(1-x_\gamma)}{\Lambda_B^2(E_\gamma)} \times \left[\left| C_9^{\rm eff}\, C_3^{SCET} + \frac{2\,C_7^{\rm eff}}{1-x_\gamma}\, C_9^{SCET}\right|^2 + \left| C_{10}\, C_3^{SCET}\right|^2\right]$$

$$\frac{d\Gamma(B^+ \to \gamma e^+ \nu)}{dE_\gamma} = \frac{\alpha G_F^2 f_B^2 |V_{ub}|^2 M_B^4}{54\pi^2} (C_3^{SCET})^2 \frac{x_\gamma(1-x_\gamma)}{\Lambda_B^2(E_\gamma)}$$

$(x_\gamma = 2E_\gamma/M_B)$

predicted ratios

$$R_q(E_\gamma) = \frac{d\Gamma(\bar{B}_q \to \gamma e^+ e^-)/dE_\gamma}{d\Gamma(B^+ \to \gamma e^+ \nu)/dE_\gamma}$$



- solid line $R_d \times 10^{-4}$
- dashed line $R_s \times 10^{-3}$
- Long distance effects (eg, $\psi^{(n)}$)?

Descotes-Genon & Sachrajda, PLB **557**, 213 (2003)

$B \to \nu\bar{\nu}\gamma$

T. Barakat, Nuovo Cim. **110A**, 631 (1997).

Y. Dinçer, arXiv:hep-ph/0204183.

$$\mathcal{L}_{\rm eff} = \frac{\alpha G_F}{\sqrt{2}\sin^2\Theta_W} V_{tb} V_{ts}^* \, C \sum_{l=e,\mu,\tau} [\bar{\nu}_l \gamma_\mu (1-\gamma_5)\nu_l]\,[\bar{s}\gamma^\mu(1-\gamma_5)b]$$

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T. Barakat, Nuovo Cim. **110A**, 631 (1997).
Y. Dinçer, arXiv:hep-ph/0204183.

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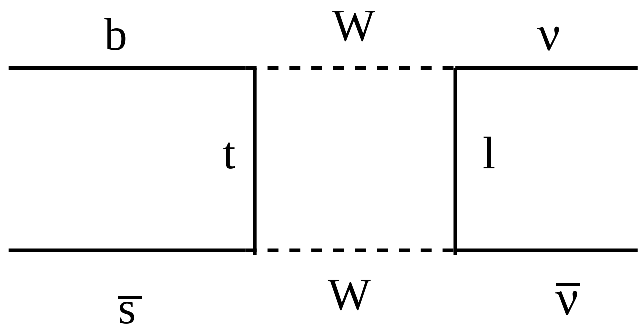

SM: $$C^{\rm SM} = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3(x_t-2)}{(x_t-1)^2}\ln x_t\right], \qquad x_t = \left(\frac{m_t}{m_W}\right)^2$$

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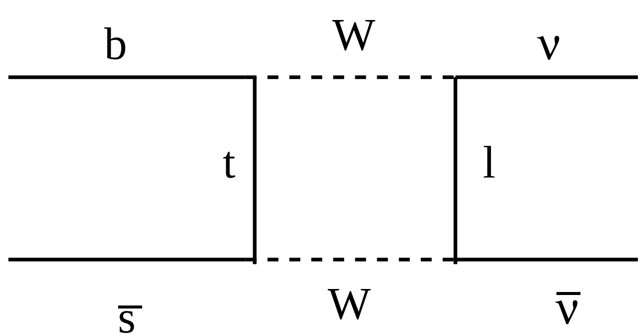

$$\sum_{\nu_i} \text{Br}(B \to \nu\bar{\nu}\gamma) \sim 7 \times 10^{-8}$$

SM: $C^{\text{SM}} = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3(x_t-2)}{(x_t-1)^2}\ln x_t\right], \qquad x_t = \left(\frac{m_t}{m_W}\right)^2$

New Physics

In models with $(V-A)\times(V-A)$ coupling only ($x = E/2M$)

$$\frac{d\Gamma}{dx}(B_s \to \nu\bar{\nu}\gamma) = \left|\frac{\alpha G_F V_{tb} V_{ts}^* C}{\sqrt{2}\sin^2\Theta_W}\right|^2 \cdot x(1-x)^3 \cdot \left[|F_A(x)|^2 + |F_V(x)|^2\right]$$

$$C = C^{\mathrm{SM}} + C^{\mathrm{new}}$$

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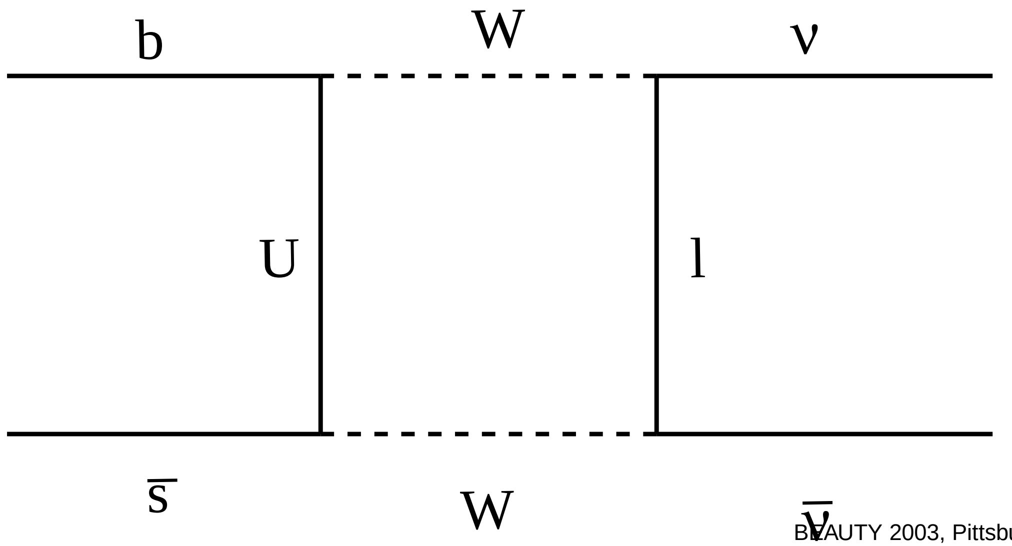

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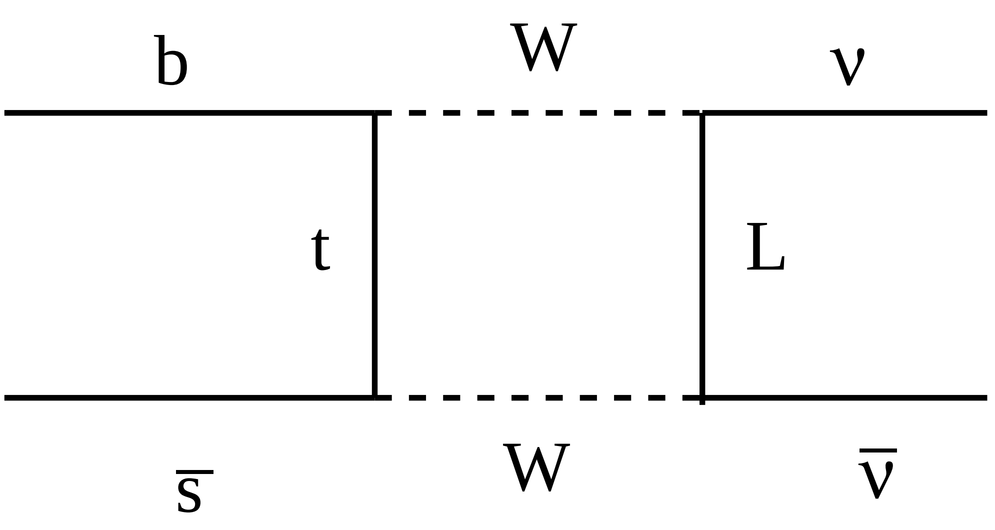
$$C=C^{\rm SM}+C^{\rm new}$$



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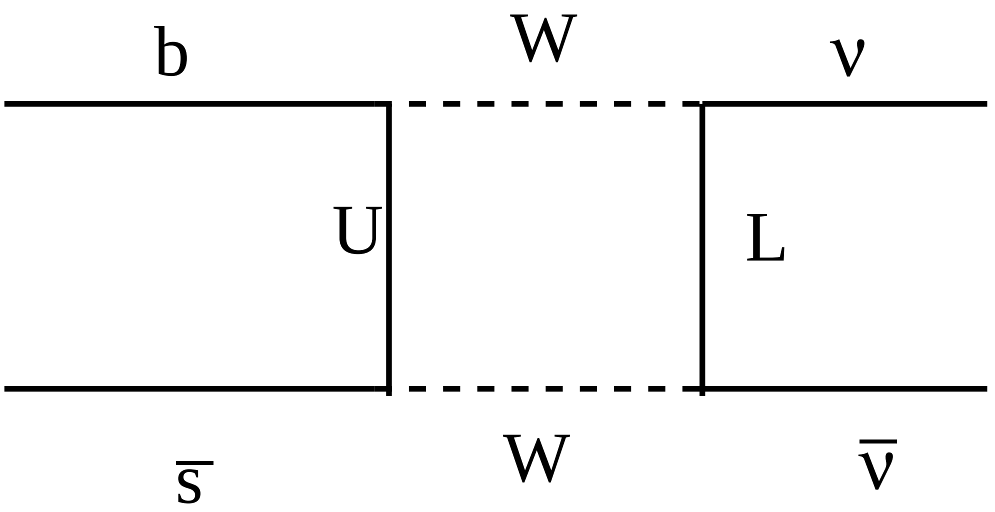
$$C = C^{\mathrm{SM}} + C^{\mathrm{new}}$$



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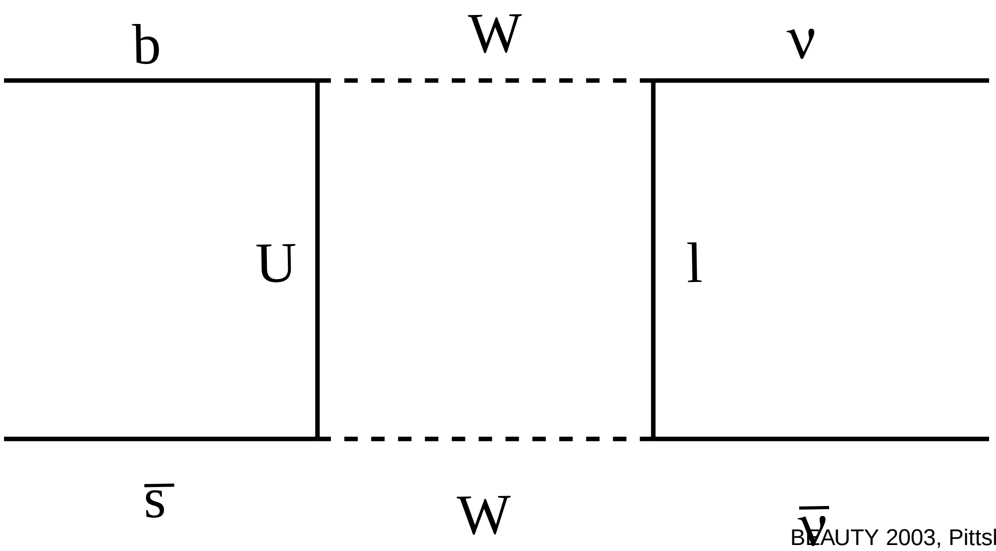
$$C = C^{\rm SM} + C^{\rm new}$$



New Physics

In models with $(V-A)\times(V-A)$ coupling only ($x = E/2M$)

$$\frac{d\Gamma}{dx}(B_s \to \nu\bar{\nu}\gamma) = \left|\frac{\alpha G_F V_{tb} V_{ts}^{*} C}{\sqrt{2}\sin^2\Theta_W}\right|^2 \cdot x(1-x)^3 \cdot \left[|F_A(x)|^2 + |F_V(x)|^2\right]$$

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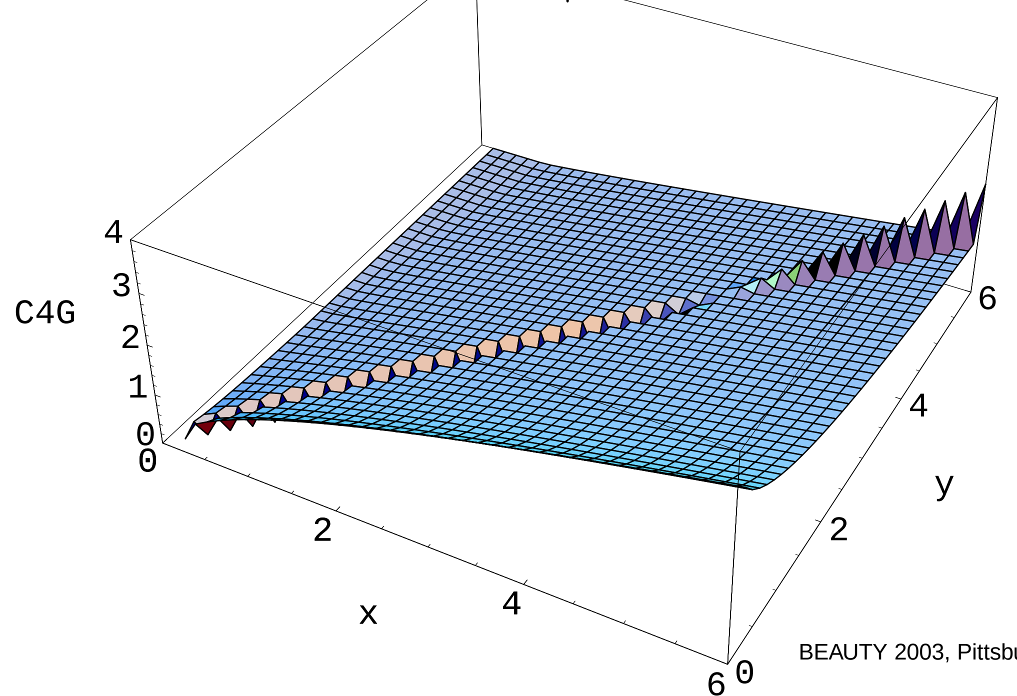
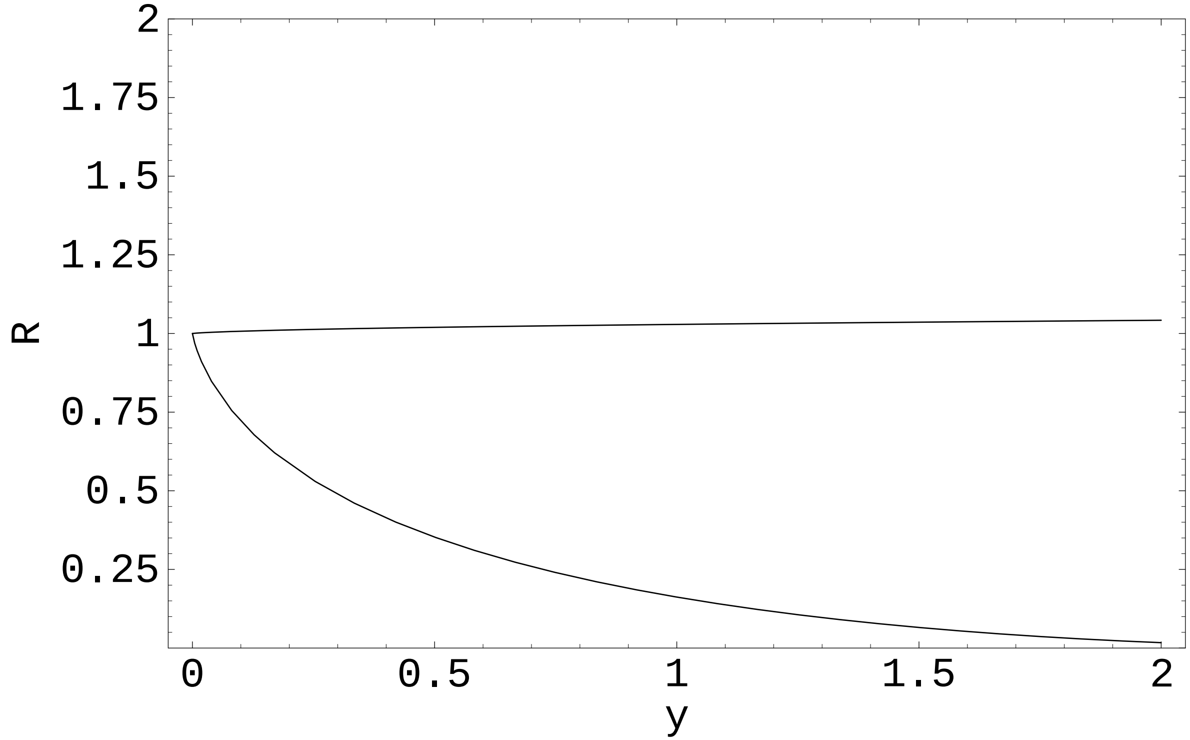
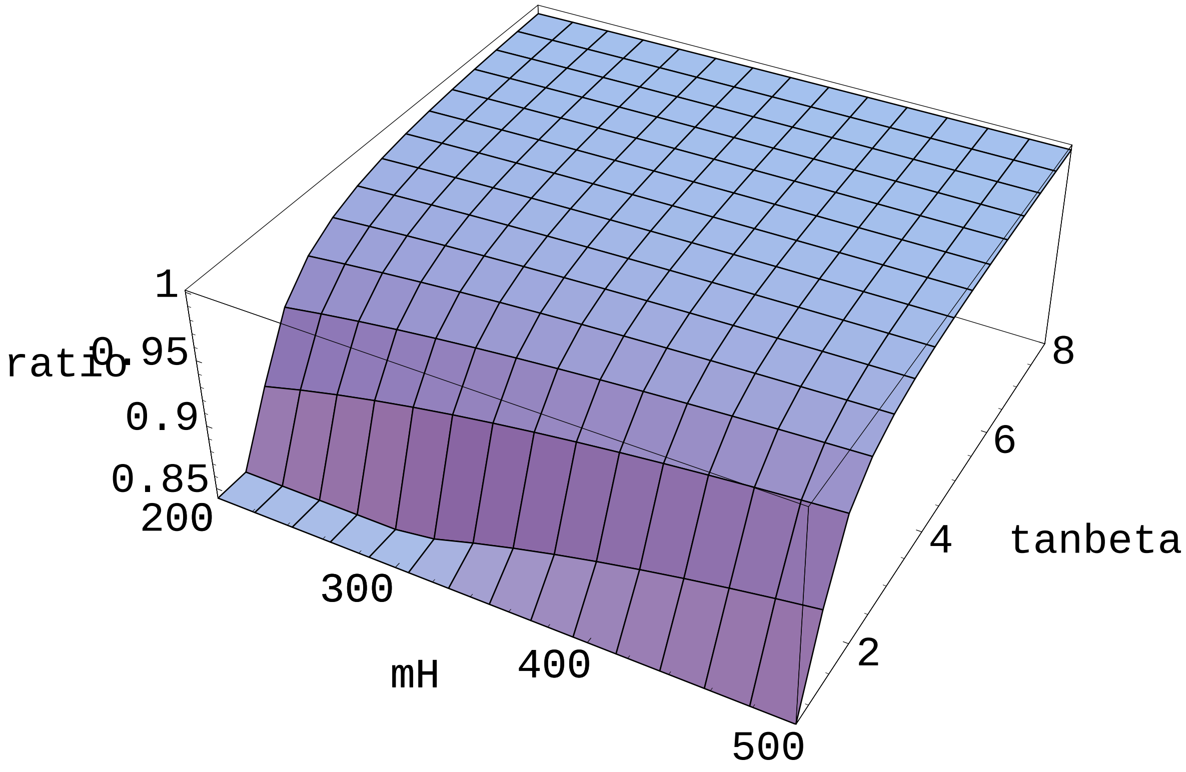


4G

$$R^{\mathrm{SM4}} := \frac{Br^{\mathrm{SM4}}(B_s \to \nu\bar{\nu}\gamma)}{Br^{\mathrm{SM}}(B_s \to \nu\bar{\nu}\gamma)}$$



2HDM



Go off-shell! $B \to \gamma^* \ell^+ \ell^-$

Y. Dincer and L. M. Sehgal, PLB**556**,169(2003)

$$\bar{B}_s \to \ell^+ \ell^- \ell'^+ \ell'^-$$

- Use amplitude for $B \to \gamma \ell^+ \ell^-$

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$$\begin{aligned}\mathcal{M}(\bar{B}_s \to l^+ l^- \gamma) &= \frac{\alpha G_F}{\sqrt{2}\pi M_{B_s}} e V_{tb} V_{ts}^* \cdot \Big[\epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} q^\rho k^\sigma (A_1 \bar{l} \gamma_\mu l + A_2 \bar{l} \gamma_\mu \gamma_5 l) \\ &\quad + i(\epsilon^*(k \cdot q) - (\epsilon^* \cdot q) k_\mu)(B_1 \bar{l} \gamma_\mu l + B_2 \bar{l} \gamma_\mu \gamma_5 l)\Big]\end{aligned}$$

$$A_1 = C_9 f_V + 2C_7 \frac{M_{B_s}^2}{q^2} f_T \,, \qquad A_2 = C_{10} f_V \,,$$

$$B_1 = C_9 f_A + 2C_7 \frac{M_{B_s}^2}{q^2} f'_T \,, \qquad B_2 = C_{10} f_A \,.$$

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- Use amplitude for $B \to \gamma \ell^+ \ell^-$
- Second $\ell'^+ \ell'^-$ pair as Dalitz pair from internal conversion
- Must distinguish double conversion in $B \to \gamma\gamma$

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$$f_V = f_A = f_T = f'_T = \frac{1}{3} \frac{f_{B_s}}{\Lambda_s} \frac{1}{x_\gamma} + \mathcal{O}(\frac{\Lambda^2_{QCD}}{E^2_\gamma})$$

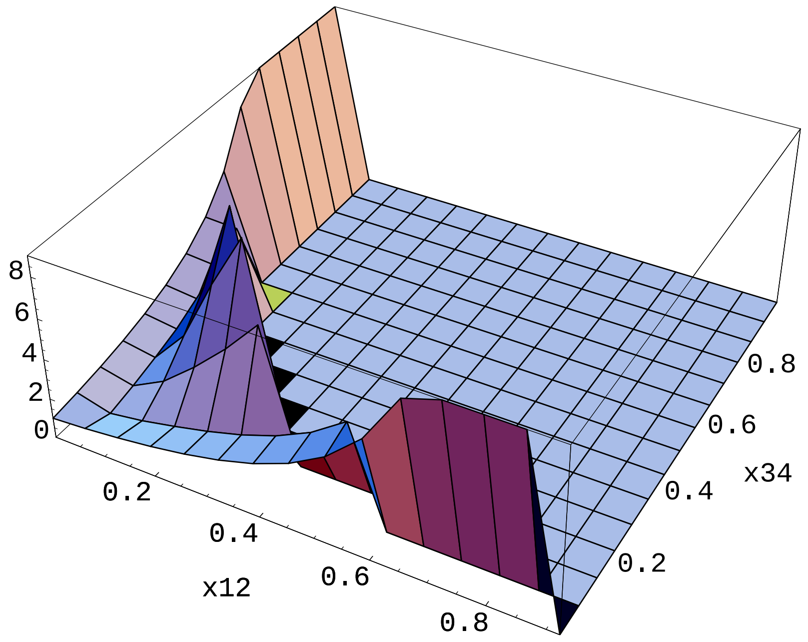
$B_s \to \ell^+\ell^-\ell'^+\ell'^-$ **numbers**

$$\mathrm{Br}(\bar{B}_s \to e\bar{e}e\bar{e}) = 3.6 \times 10^{-10}\,,$$

$$\mathrm{Br}(\bar{B}_s \to e\bar{e}\mu\bar{\mu}) = 1.1 \times 10^{-10}\,,$$

$$\mathrm{Br}(\bar{B}_s \to \mu\bar{\mu}\mu\bar{\mu}) = 3.5 \times 10^{-11}\,.$$

Ratio $(d\Gamma/dx_{12}dx_{34})_{EW}/(d\Gamma/dx_{12}dx_{34})_{2xDalitz}$



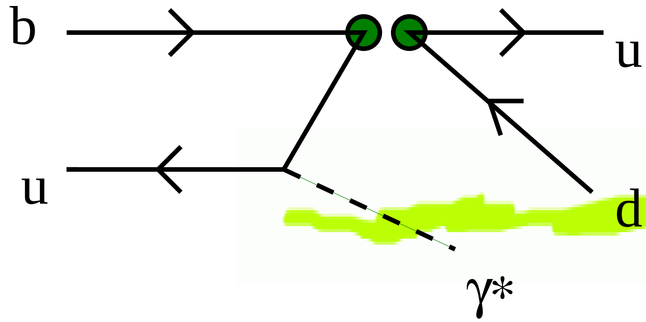
$x_{ij} \equiv (q_i + q_j)^2/m_B^2$

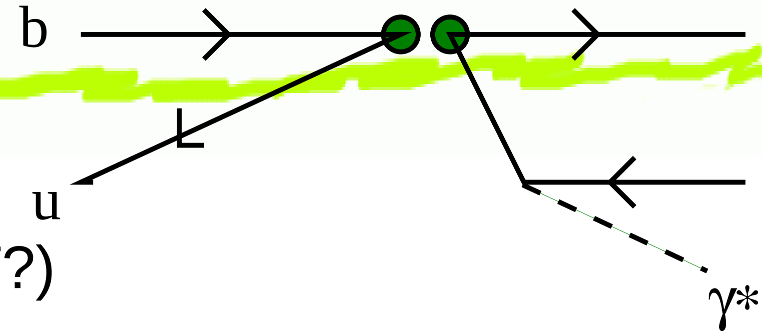
$B \to \pi \ell^+ \ell^-$

B.G., D. R. Nolte and I. Z. Rothstein, PRL **84**, 4545 (2000)

- Two amplitudes
  - Short distance: "known" form factors
  - Long distance (W-exchange plus γ^* emission) computable!
- Interference $\Rightarrow \cos\alpha$

Long Distance

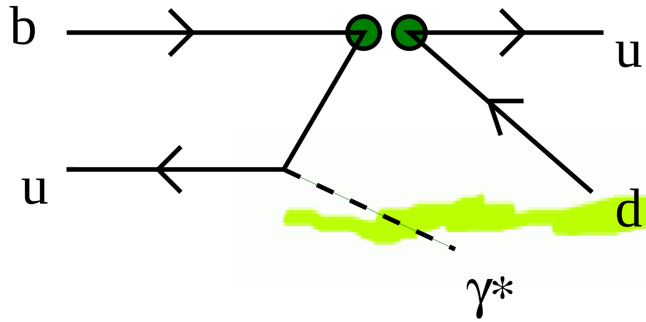


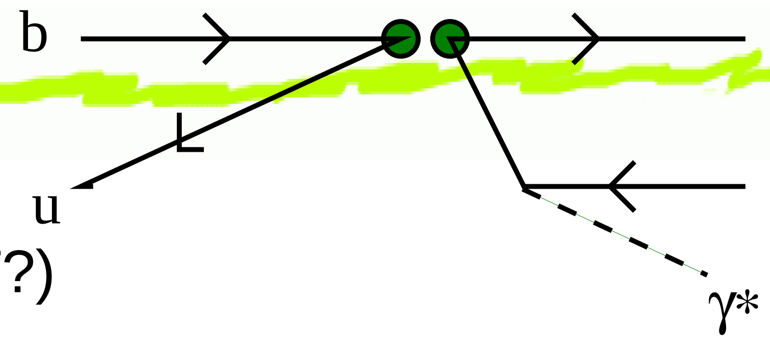


- Amplitude Factors (SCET?)

$$H^\mu = \langle \pi | \int d^4x \; e^{iq\cdot x} \; T(j^\mu_{\rm em}(x)\mathcal{H}'_{\rm eff}(0))|B\rangle.$$

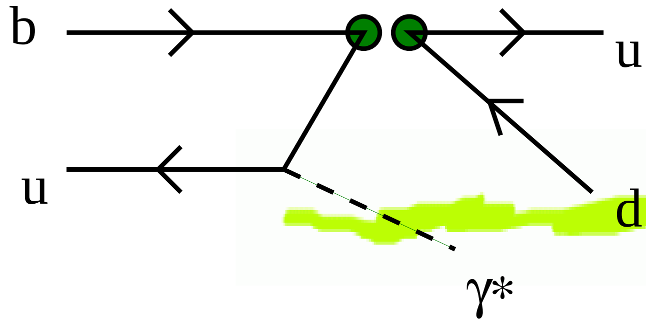
Long Distance



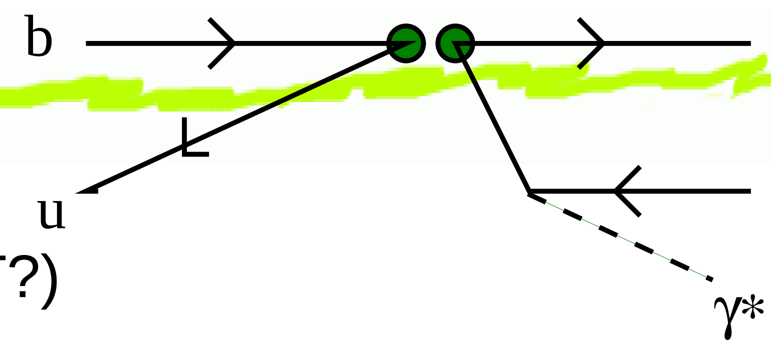


- Amplitude Factors (SCET?)

$$H^\mu = \kappa \int d^4x\, e^{iq\cdot x} \Big[\langle \pi |\, T(j^\mu_{\rm em}(x) j_\lambda(0)) |0\rangle \tfrac{1}{2} f_B p_B^\lambda + \langle \pi | j_\lambda(0) |0\rangle \langle 0 | T(j^\mu_{\rm em}(x) J^\lambda(0)) |B\rangle \Big]$$



Long Distance



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- First line: Ward identity $\Rightarrow -e\kappa f_\pi f_B p_B^\mu$
- Second line: Off-shell gamma-FF
  - Expansion in $\Lambda_{\rm QCD} m_b/q^2$
  - HQET $\Rightarrow$ first correction too!

$$H^\mu = -\tfrac{4}{3} e\kappa f_\pi f_B p_B^\mu (1 + \tfrac{2}{3}\bar\Lambda m_b/q^2)$$

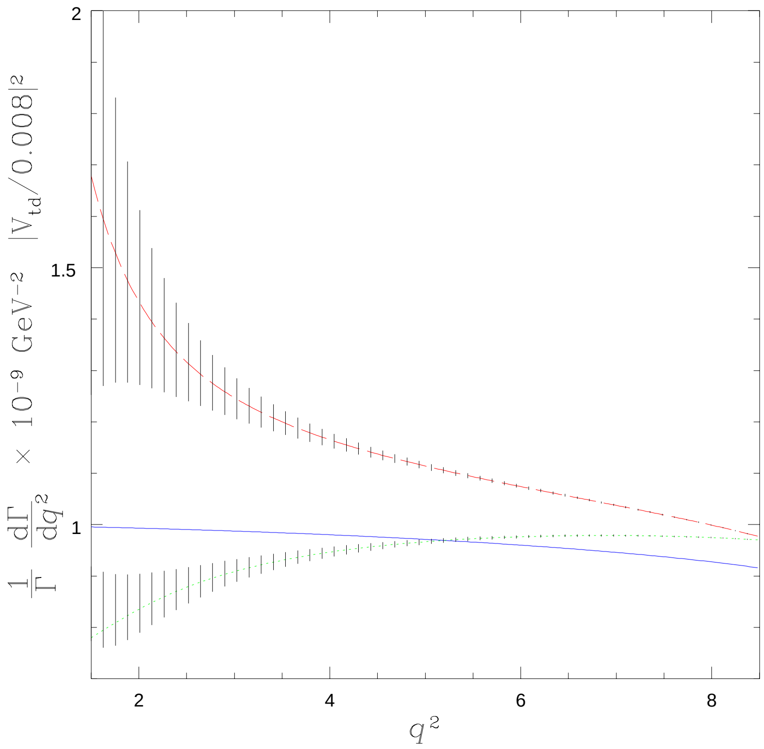
Short Distance -same old

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{d}\sigma^{\mu\nu} P_+ b) F_{\mu\nu} \qquad \mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2} (\bar{d}\gamma^\mu P_- b) \bar{e}\gamma_\mu(\gamma_\mu\gamma_5)e$$

$$\langle \pi(p')|\bar{d}\gamma^\mu b|B(p)\rangle = (p+p')^\mu f_+ + (p-p')^\mu f_- \qquad \langle \pi(p')|\bar{d}\sigma^{\mu\nu} b|B(p)\rangle = 2ih(p^\nu p'^\mu - p^\mu p'^\nu)$$

$$\frac{d\Gamma}{dq^2} = |V_{tb}V_{td}^*|^2 \frac{G_F^2 \alpha^2 m_B^3}{3\times 2^9 \pi^5} \frac{(m_B^2 - q^2)^3}{m_B^6} \Bigg[|C_{10} f_+|^2 + \left| \tilde{C}_9 f_+ + 2 m_b C_7 h - \frac{16\pi^2}{3} \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \frac{c(m_b) f_\pi f_B}{q^2} (1 + \frac{2\bar{\Lambda} m_b}{3q^2}) \right|^2 \Bigg]$$

$\cos\alpha$?



solid $\cos\alpha = 0$

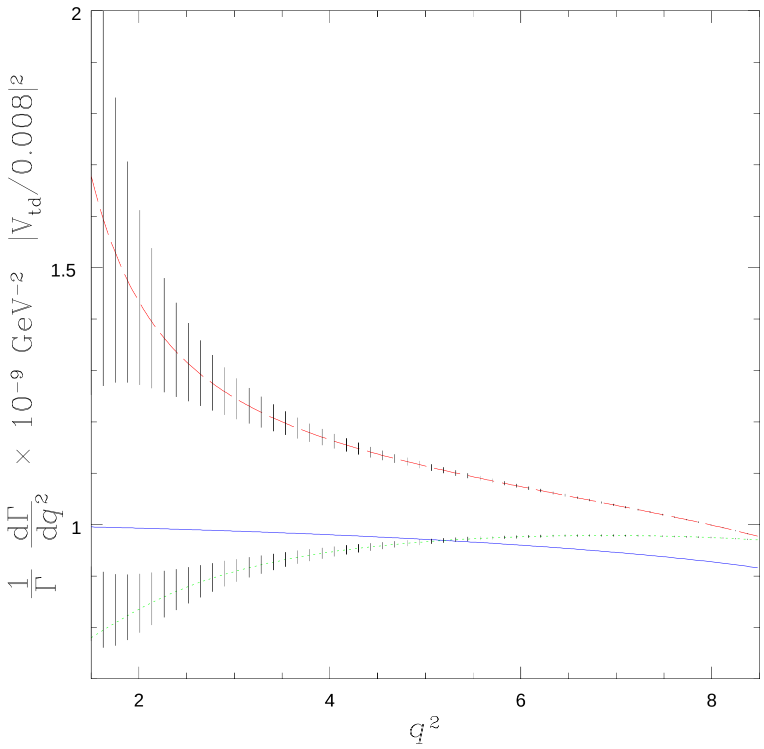
dashed $\cos\alpha = -1$

dotted $\cos\alpha = +1$

shaded $=$ parametric error

$q^2 < m^2_{J/\psi}$

$\cos\alpha$?



solid $\cos\alpha = 0$

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As $q^2 \to 0$, back to on-shell FF $\Rightarrow$ extend prediction?

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- Multitude of “ultra”-rare processes

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- Two lessons:
  - Many with same one form factor: $B \to \gamma\gamma$, $B \to \gamma\ell\nu_\ell$, $B \to \gamma\ell^+\ell^-$, $B \to \nu\bar{\nu}\gamma$
  - Accidental suppression in SM is good: new physics sensitivity

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  - Accidental suppression in SM is good: new physics sensitivity
- Out of time:
  - “Standard:” $b \to s\nu\bar{\nu}$, $B \to \mu^+\mu^-$, $\ldots$
  - Less convenional: $B \to \mu^+e^-$, $B^- \to D^+e^-e^-$, $\ldots$

$B^- \to D^+ e^- e^-$

BG, unpublished

- Sensitive to different masses/energies/species than nuclear-2β or $K^- \to \pi^+ \ell^- \ell^-$
- Fun QCD:
  - Semi-inclusive calculable! Same rigour as semileptonic. Experimentally impossible

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- Fun QCD:
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  - Exclusive $B^- \to D^+ e^- e^-$, same matrix element as in $\Delta\Gamma$(using HQET symmetries)
  - Exclusive $B \to D\pi e^- e^-$, same as in $B \to D\pi$, factorizes, cleanly computable.
- Ultra^3 in SM
- Much larger in extensions, eg, R-parity breaking