Lattice QCD Now and in > 5 Years

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Outline:

- Motivation
- Introduction to lattice QCD
- Some recent developments
- Prospects for the near future
- f_B
- Semileptonic B meson decays
  - $B \to D, D^* \, l\nu$
  - $B \to \pi \, l\nu$
- Issues
- Conclusions & Outlook

Motivation

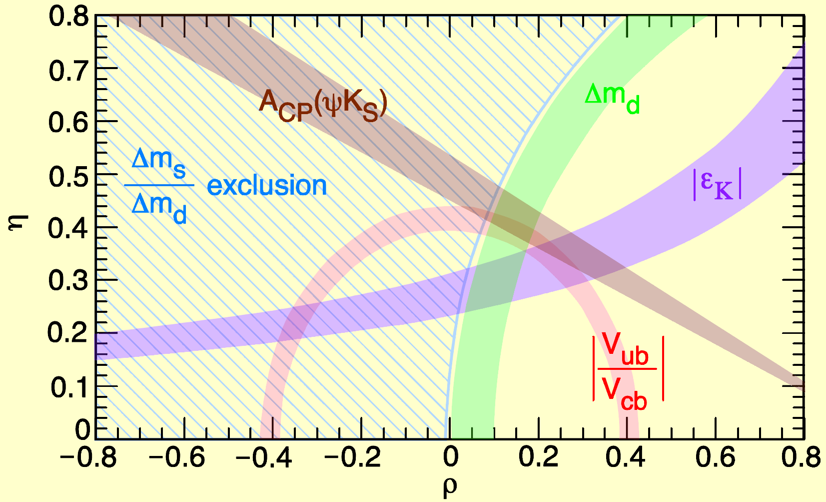
The problem:

for example
$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = (known) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

need the hadronic matrix elements from lattice QCD
to determine the CKM matrix elements

goal:

2-3% theory errors
from lattice QCD

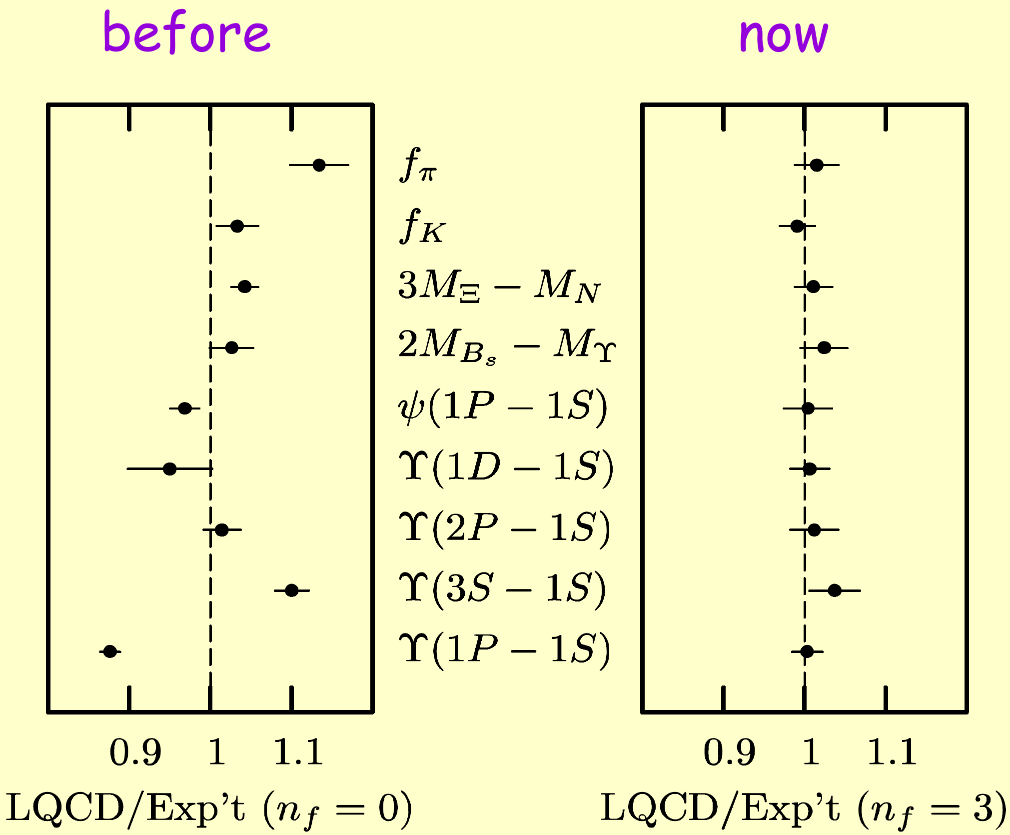


(CLEO-c)

Motivation cont'd

HPQCD (NRQCD+MILC+FNAL), compiled by P. Lepage (hep-lat/0304004)

lattice QCD/experiment



works quite well!

Introduction to Lattice QCD

in general: $\langle \mathcal{O} \rangle^{\text{lat}} = \langle \mathcal{O} \rangle^{\text{cont}} + O(ap)^n \qquad n \geq 1$

errors scale with the typical momenta of the particles,
e.g. $(\Lambda_{QCD}\, a)^n$ for gluons and light quarks. keep $1/a$ π Λ_{QCD}

$\Lambda_{QCD} \sim 200 - 300$ MeV
typical lattice spacing $a \sim 0.1$ fm $\Rightarrow 1/a \sim 2$ GeV

Improvement: add more terms to the action to make n large

- light quarks (m_q Λ_{QCD} and am_q 1):
  - staggered (Kogut+Susskind): a^2 errors
    improved staggered (Asqtad): $\alpha_s\, a^2$ errors (Lepage, MILC)
  - Wilson: a errors ($n = 1$)
    Clover (SW): $\alpha_s^2\, a$ errors, a^2 errors (Sheikholeslami+Wohlert)
  - Domain Wall fermions, Overlap, …

Introduction to Lattice QCD, cont'd

- Heavy Quarks (m_Q π Λ_{QCD} and am_Q ϕ 1):

  lattice NRQCD (Lepage, et al., Caswell+Lepage) :

  - discretize NRQCD lagrangian: valid when $am_Q > 1$
  - errors: ~ $(ap)^n$, $(p/m_Q)^n$

  Fermilab (Kronfeld, Mackenzie, AXK):

  - rel. Wilson action has the same heavy quark limit as QCD
  - add improvement: preserve HQ limit
  - smoothly connects light and heavy mass limits, valid for all am_Q
  - errors: ~ $(ap)^n$, $(p/m_Q)^n$


s. harris
IF THERE WERE COMPUTERS IN GALILEO'S TIME

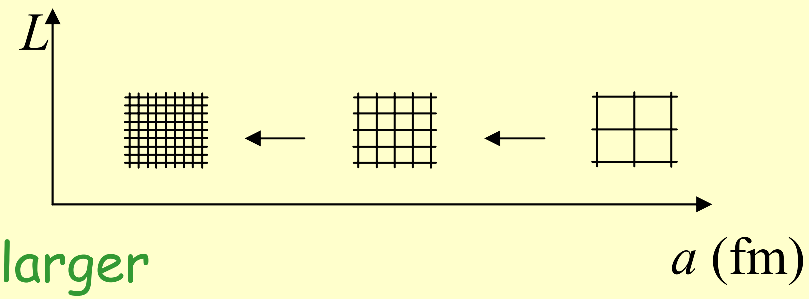
errors, errors, errors, …

✓statistical errors: from monte carlo integration

✓ finite lattice spacing, a: $\langle \mathcal{O} \rangle^{lat} = \langle \mathcal{O} \rangle^{cont} + O(ap)^n$

take continuum limit:

•brute force:
computational effort grows
like ~ $(L/a)^6$



•improving the action: make n larger

✓ finite volume

➢ m_l dependence: chiral extrapolation

➢ n_f dependence: sea quark effects

❖ perturbation theory $\langle J_\mu^{\rm cont} \rangle = Z^{\rm lat} \langle J_\mu^{\rm lat} \rangle$

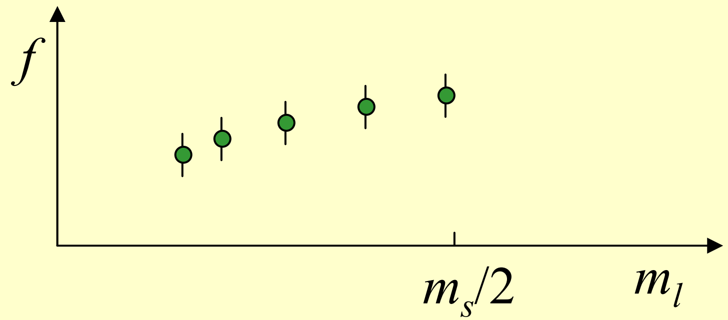
systematic errors, cont'd

- chiral extrapolation, m_l dependence:

In numerical simulations, $m_l > m_{u,d}$ because of the computational cost for small m.

use chiral perturbation theory to extrapolate to $m_{u,d}$

need $m_l < m_s/2$ and several different values for m_l (easier with staggered than Wilson-type actions)

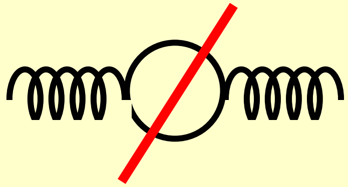


Decay constants, form factors:

chiral logs contribute $\sim m_\pi^2 \log(m_\pi^2)$

systematic errors, cont'd

- n_f dependence

$\boldsymbol{n_f = 0}$:  quenched approximation

introduces systematic error ~ 10 – 30 % for stable hadrons

$\boldsymbol{n_f \neq 0}$: computationally difficult

keep a large, $a \geq 0.1$ fm ⇒ need improved actions

until 2002: $n_f = 2$ with staggered and SW fermions (a^2 errors)

new MILC (2002):

$\boldsymbol{n_f = 3}$ with $m_s \neq m_{light}$ and $m_{light} = m_s/8,\ m_s/4\ \ldots\ ,\ m_s/2, \ldots,\ m_s$

using an improved staggered action ($\alpha_s a^2$ errors)

"Gold-Plated" Quantities
or
What are the "easy" lattice calculations ?

For stable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- π, K, D, D_s, B, B_s mesons
  masses, decay constants, weak matrix elements for mixing, semileptonic and rare decays

- charmonium and bottomonium (η_c, J/ψ, h_c, …, η_b, $Y(1S)$, $Y(2S)$, ..) states below open D/B threshold
  masses, leptonic widths, electromagnetic matrix elements

This list includes most of the important quantities for CKM physics. Excluded are ρ mesons and other resonances.

gold-plated quantities for most CKM elements …

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
 & K \to \pi l \nu & B \to \pi\ l \nu \\
V_{cd} & V_{cs} & V_{cb} \\
D \to \pi l \nu & D \to K l \nu & B \to D, D^* l \nu \\
D \to l \nu & D_s \to\ l \nu & \\
V_{td} & V_{ts} & V_{tb} \\
B^0 - \overline{B^0} \text{ mixing} & B_s - \overline{B_s} & \\
K^0 - \overline{K^0} & &
\end{pmatrix}
$$

Recent Developments

- the new MILC configurations include realistic sea quark effects.

  strategy:

  the only free parameters in lattice QCD lagrangian:
  quark masses and α_s

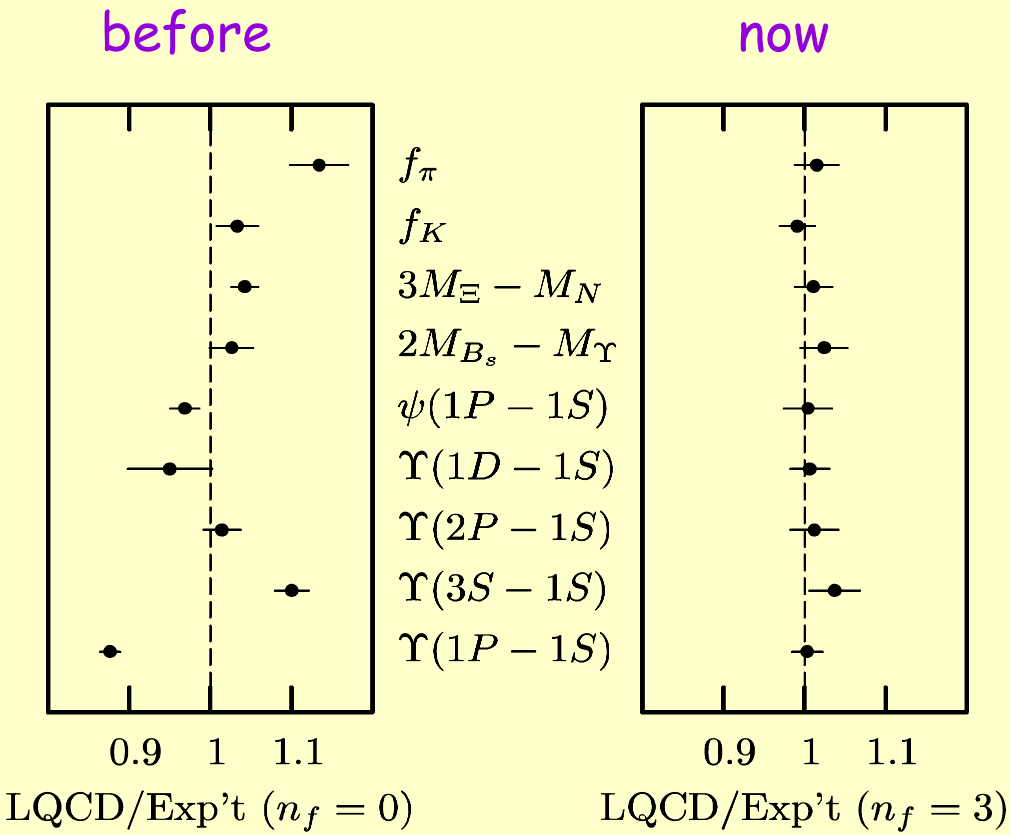
  tune the lattice QCD parameters using experiment:
  $m_{u,d}$, m_s , m_c , m_b using π, K, D_s, Y meson masses
  α_s using 2S-1S splitting in Y system

  all other quantities should agree with experiment …
  try this for some easy quantities …

Recent Developments cont'd

HPQCD (NRQCD+MILC+FNAL), compiled by P. Lepage (hep-lat/0304004)

lattice QCD/experiment



agreement within ~few % (stat+sys.) errors

Prospects for the near future

work currently in progress using the MILC configurations
within the next year we can expect first results for …

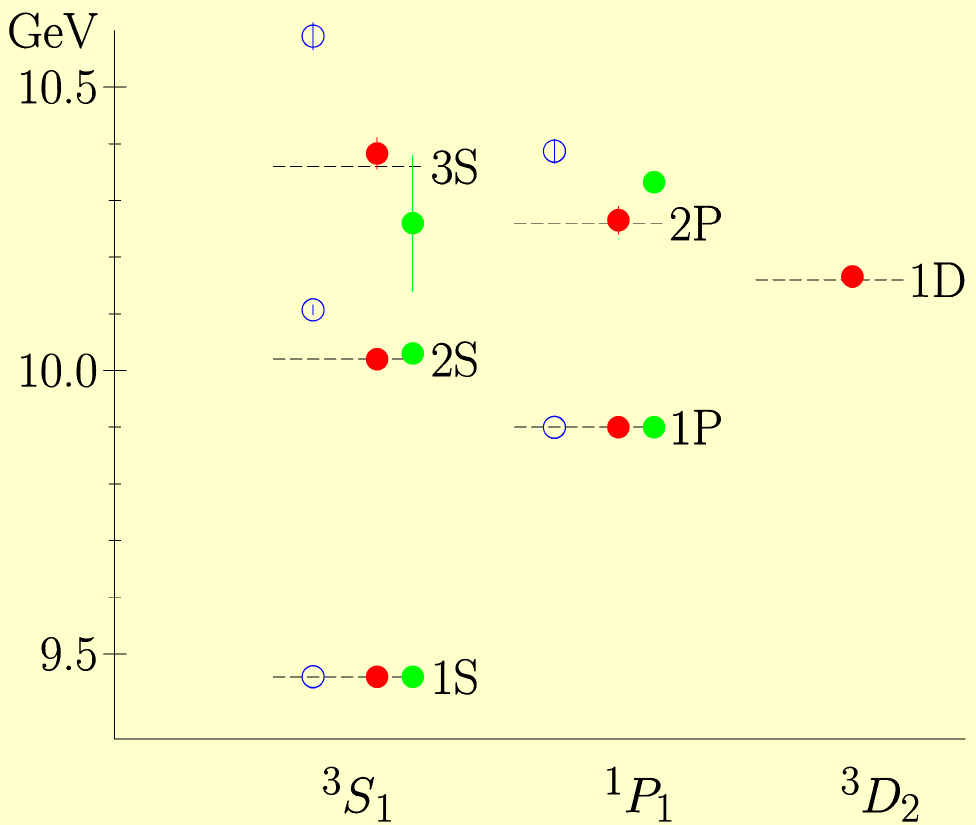
- ✓ Υ and J/ψ systems using NRQCD and Fermilab actions

  test the new heavy quark actions

  $\Rightarrow \alpha_s,\ m_b,\ m_c$

The Upsilon Spectrum

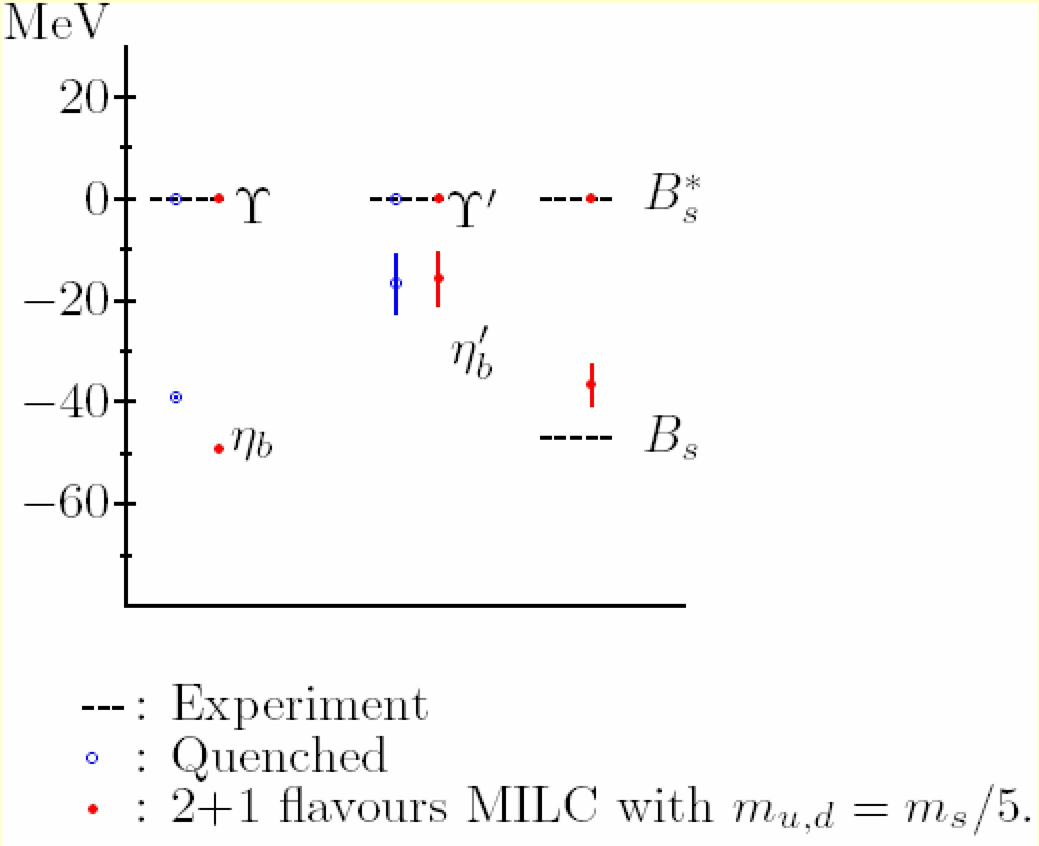
HPQCD (Davies, Gray, et al) 2003



○ Quenched ● $n_f = 2+1$, $a = 0.13$ fm (MILC)
● $n_f = 2+1$, $a = 0.09$ fm (MILC)

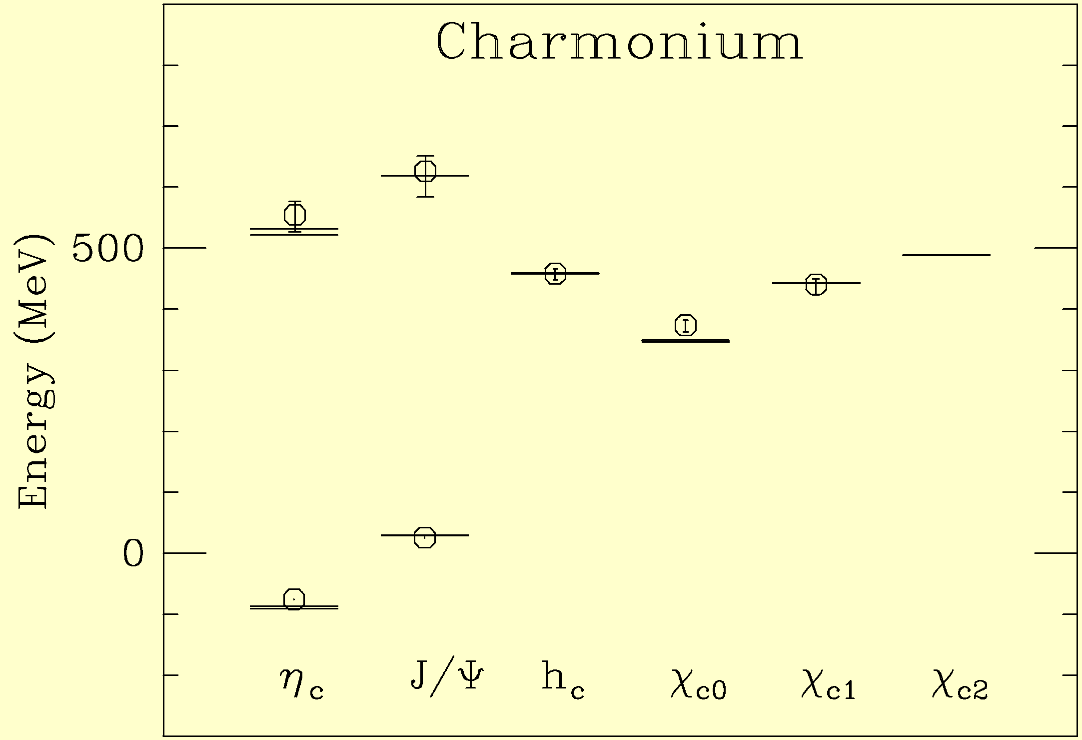
The Upsilon Hyperfine Spectrum

HPQCD (Davies, Gray, et al) 2003



for hyperfine splittings still need 1-loop correction to coefficient of $\sigma \cdot \boldsymbol{B}$ term

Fermilab 2003 (preliminary)



result at $m_{light} = m_s/4$, $a = 0.12$ fm with $O(a)$ improved action

Prospects for the near future cont'd

… and expect new results for …

✓ D, D_s, B, B_s meson systems (NRQCD, Fermilab)

using improved staggered light quarks with $m_l < m_s/2$

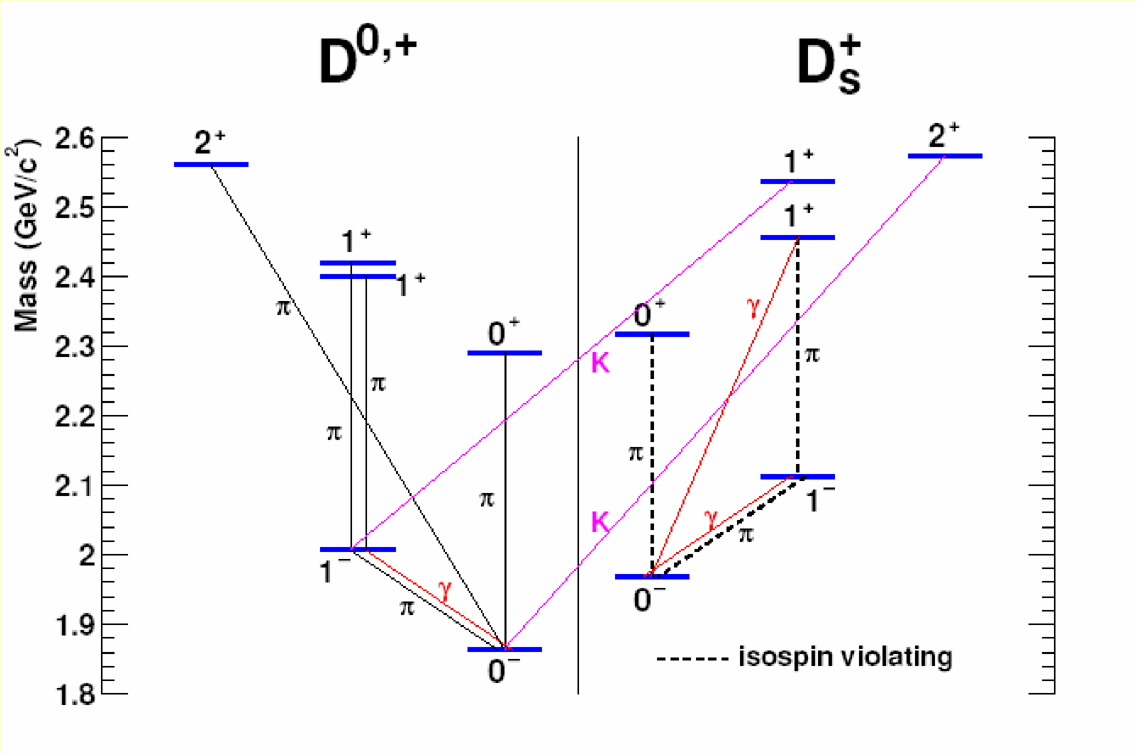
masses (splittings), decay constants, mixing, SL form factors

⇒ comparison with CLEO-c essential to test lattice results

expect initial accuracy of < 10% errors with an ultimate goal of 2-3% errors.

The D_s Spectrum

Recent experimental surprise in the D_s spectrum (BaBar, CLEO, Belle):



0^+ and 1^+ states are too low to decay into $D^{(*)}K$ final states

$\Rightarrow$ they decay into $D_s^{(*)}\pi$; violates Isospin

$\Rightarrow$ states have very narrow widths

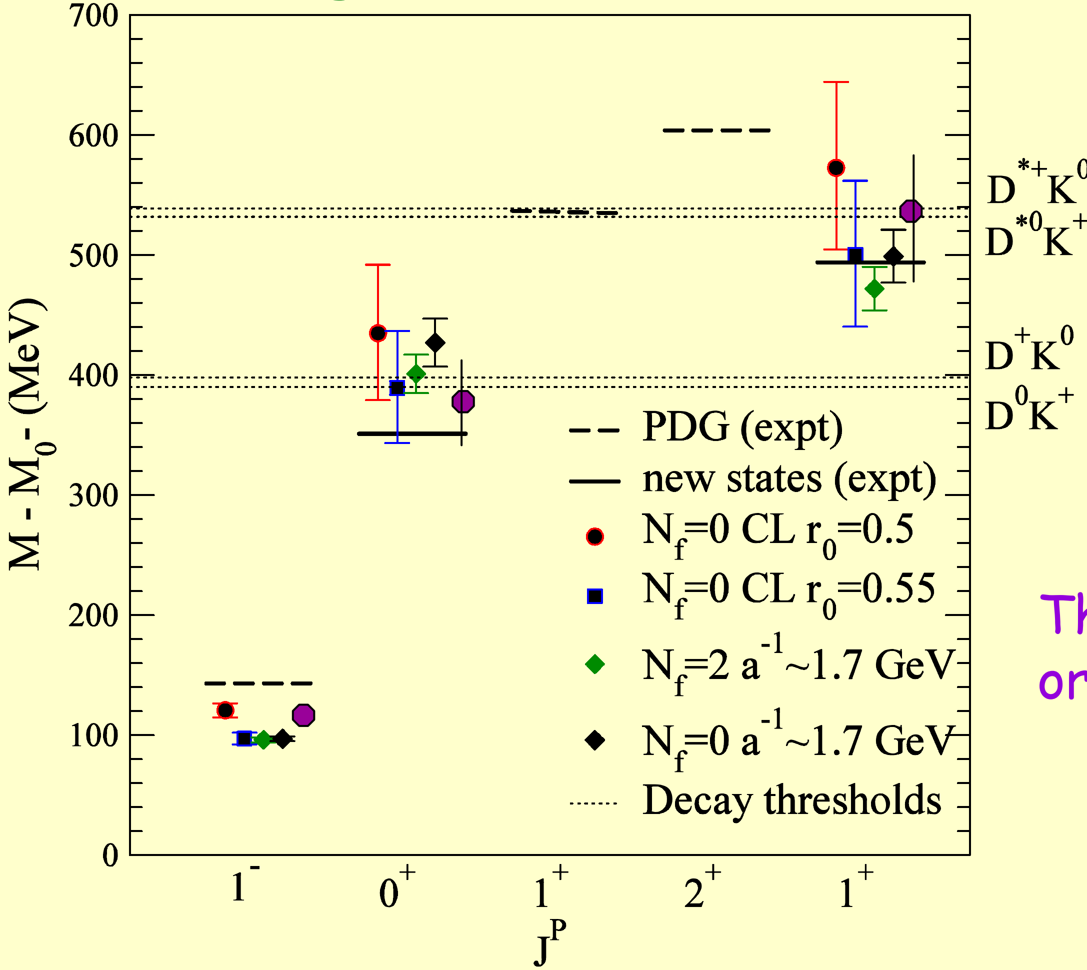
The new 0^+ and 1^+ states are close (~50 MeV) to $D^{(*)}K$ threshold

$\Rightarrow$ they are not gold-plated quantities for LQCD, may have significant threshold effects

The D_s Spectrum cont'd

UKQCD (Dougall, et al) 2003 + FNAL (Mackenzie, et al) 2003

preliminary



$n_f = 2+1$, $a = 0.13$ fm (MILC)

The new states look like ordinary P-wave cs states.

f_B

- $n_f = 0$: $f_B = 173\,(23)$ MeV (Yamada average at Lattice 2002)
  has been stable in the last four years
  dominant error: n_f dependence
  new in 2003: f_{Bs} with nonpert. matching (APE, ALPHA)

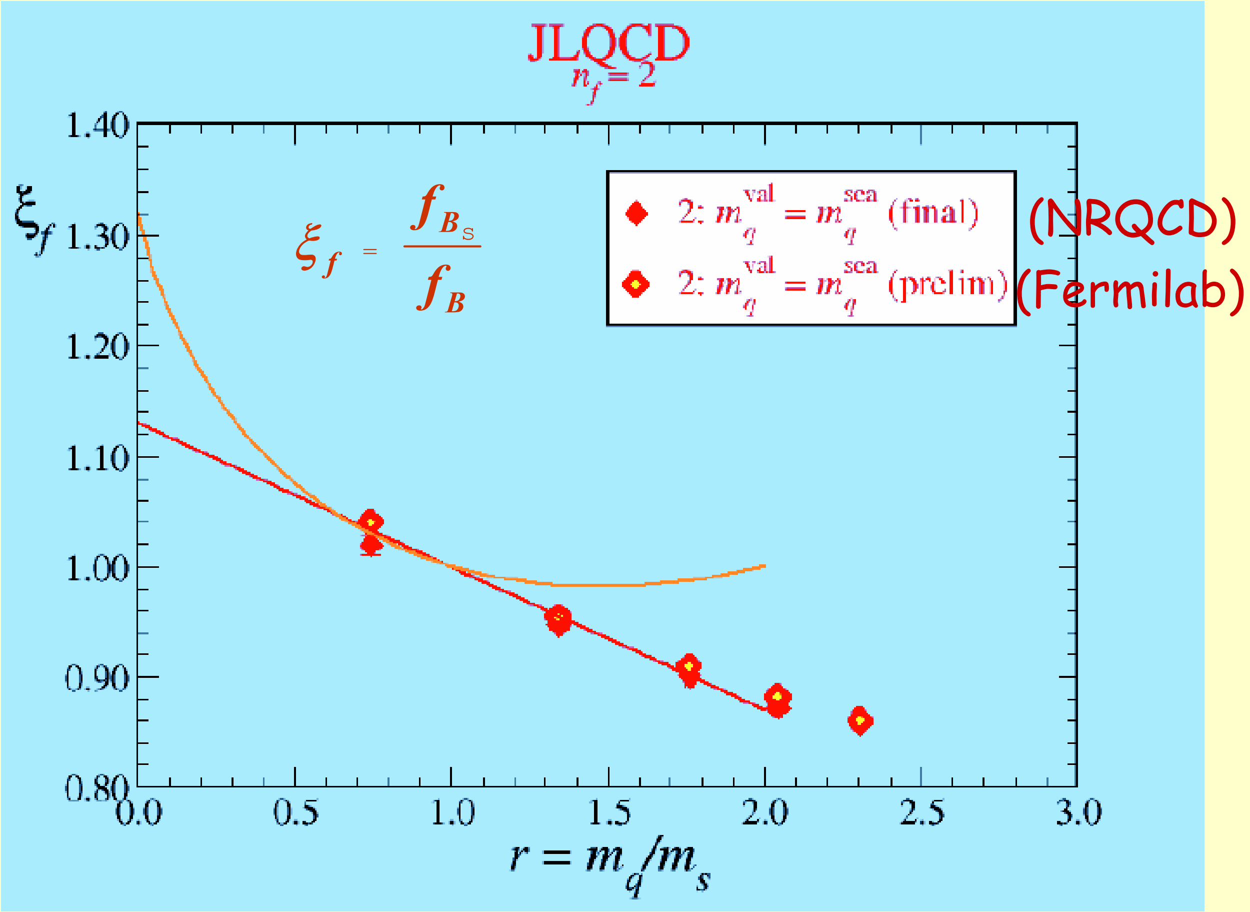
- $n_f \neq 0$: most results (< 2003) have $n_f = 2$
  "heavy" (valence) light quarks with $m_l \geq m_s/2$
  $\xi_f = f_{Bs} / f_{Bd} = 1.16\,(5)$ agrees with $n_f = 0$

- in 2002:
  include chiral logs in chiral extrapolation (JLQCD)
  increases $f_{Bs} / f_{Bd} \rightarrow 1.3$
  increases the systematic error due to m_l dependence
  1st result with $n_f = 3$ (MILC, Lattice 2002) but also with "heavy" valence light quarks

- new in 2003:
  preliminary results from HPQCD & FNAL (Lattice 2003)
  for decay constants on MILC lattices

chiral logarithms

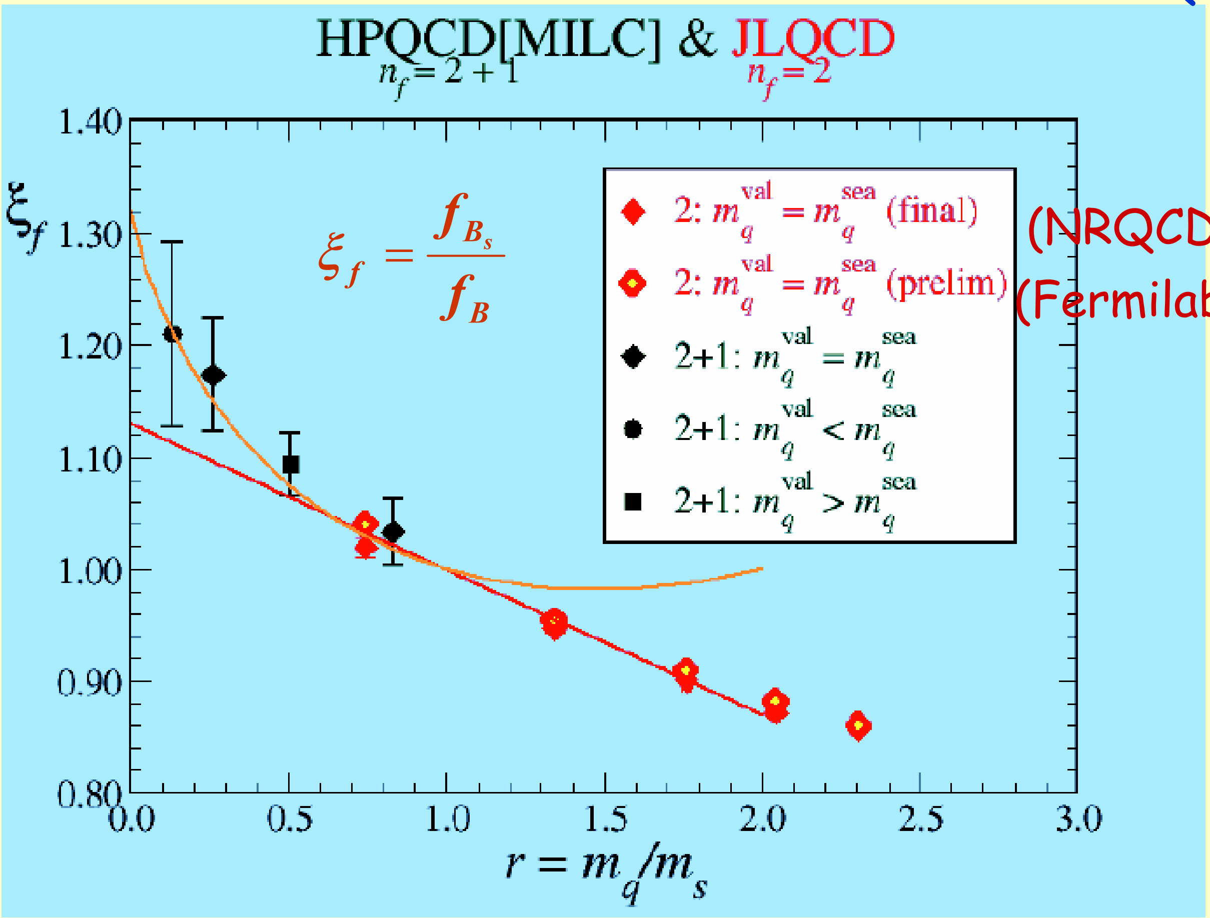
Kronfeld review @ Lattice 2003



chiral logarithms

Kronfeld review @ Lattice 2003

HPQCD: PRELIMINARY



(NRQCD)

(Fermilab)

This yields for

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$\xi = 1.23\ (5)$ JLQCD

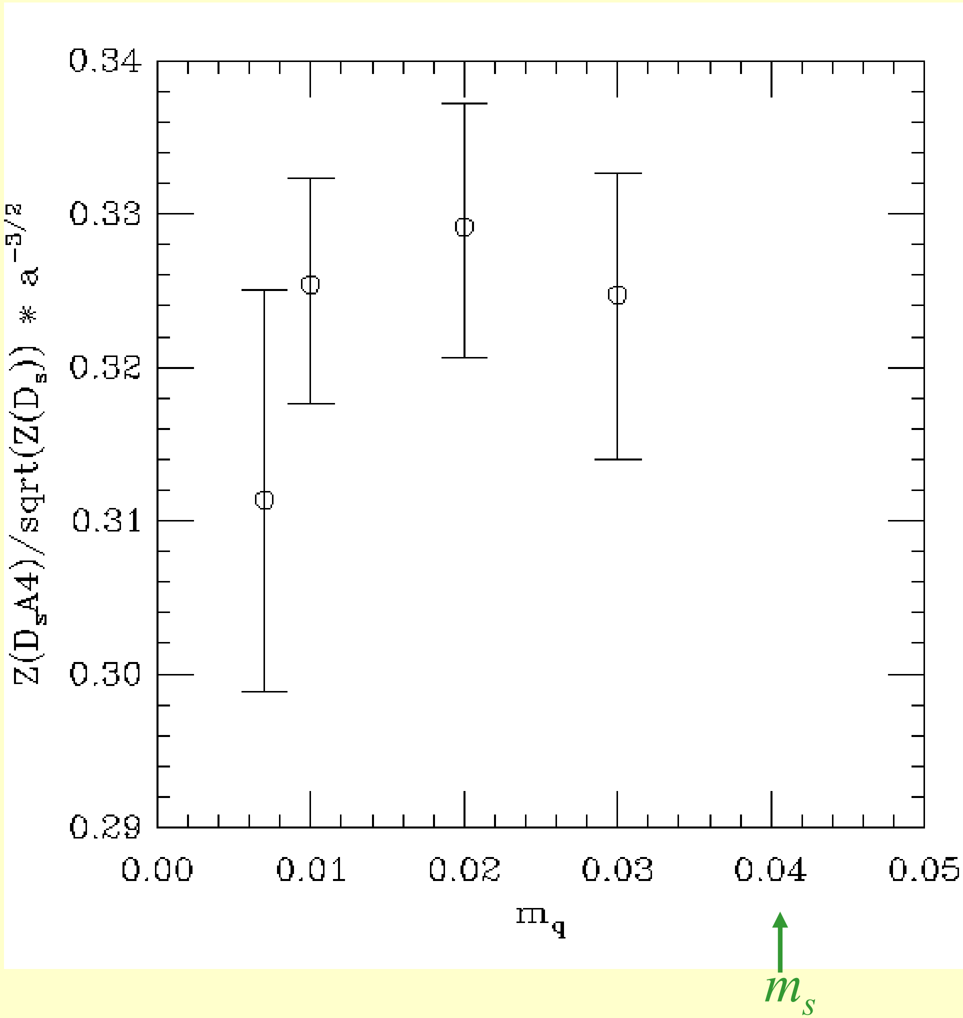
$1.32\ (5)$ HPQCD

$g^2 \sim 0.35$ (CLEO)

- Kronfeld+Ryan: chiral logs are small for B_B
- Becirevic, et al: use double ratios: $(f_{Bs}/f_{Bd})/(f_K/f_\pi)$

f_{Ds} vs sea quark mass

FNAL (Mackenzie, et al) @ Lattice 2003



PRELIMINARY

f_{Ds} : +/- few % (stat)
+/- $O(\alpha_s)$ (pert. thy)

pert. theory (1-loop):
in progress
(Nobes, Trottier,AXK)

+ new results from HPQCD for f_{Ds}

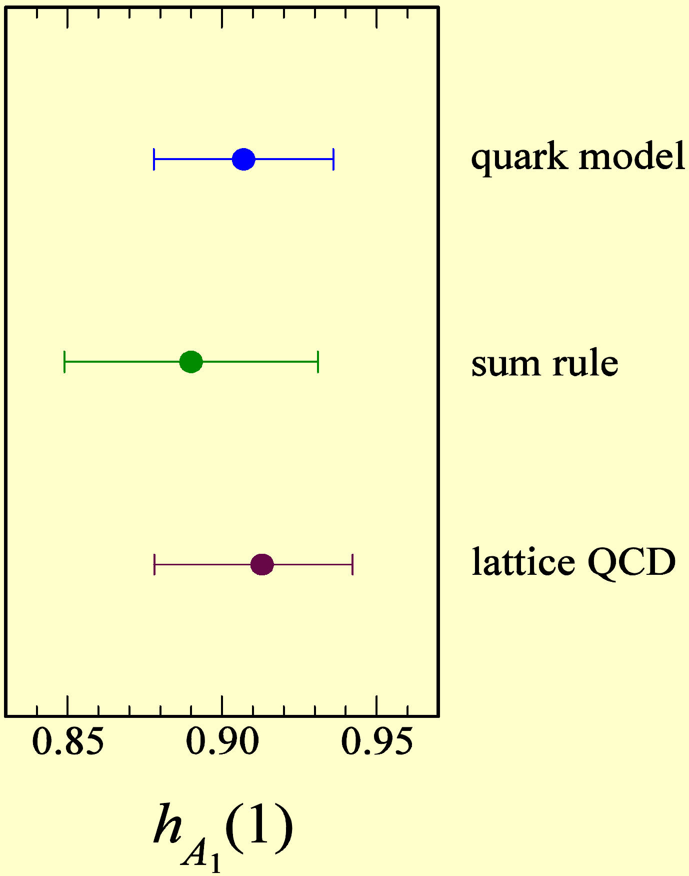
$B \to D, D^* l\nu$

Hashimoto, et al (FNAL):

$$\mathcal{F}_{B\to D}(1) = 1.058 \pm 0.016^{+0.014}_{-0.005}$$

$$\mathcal{F}_{B\to D^*}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003+0.000+0.006}_{-0.014-0.016-0.014}$$

stat. pert. thy a m_l n_f



- FNAL result is obtained at $n_f = 0$
- $n_f=3$ will be done soon
- chiral extrapolation error will be reduced by having $m_l < m_s/2$

$B, D \rightarrow \pi l \nu$

- $p_\pi\,(q^2)$ dependence: $\hat{p}_\pi \neq 0$

$$\langle \pi \,|\, V_\mu \,|\, B \rangle^{\mathrm{lat}} = \langle \pi \,|\, V_\mu \,|\, B \rangle^{\mathrm{cont}} + O(ap_\pi)^n$$

$$\Rightarrow \quad p_\pi\,\delta\;\; 1\,\mathrm{GeV}$$

improved actions help (keep n large)

- experiment: measure $d\Gamma/dp_\pi$ for $0 \;\bar{}\; p_\pi < m_B/2$

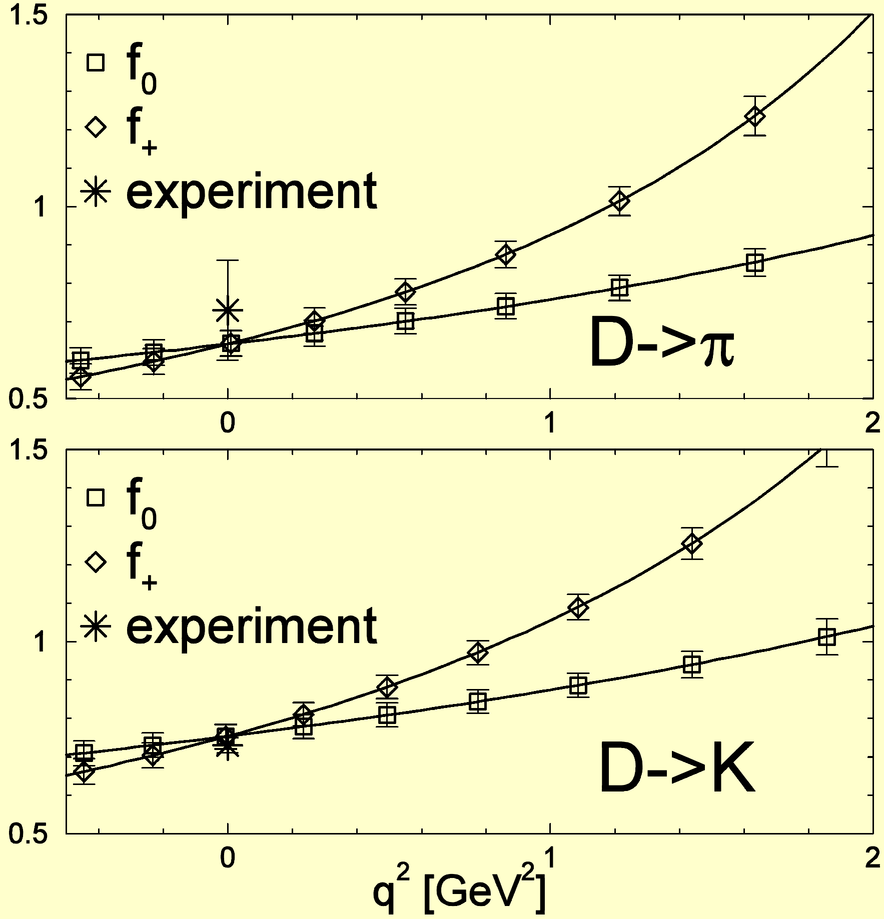
- Note: not a problem for D decays

- Prior to 2003: $n_f = 0$ only

- New in 2003:
  new (preliminary) results for ***B, D*** decays to light hadrons
  on MILC lattices (FNAL, MILC)

$D \to \pi, K\, l\nu$

FNAL (Okamoto, et al) @ Lattice 2003 on the MILC ($n_f = 2+1$, $a = 0.13$ fm) lattices

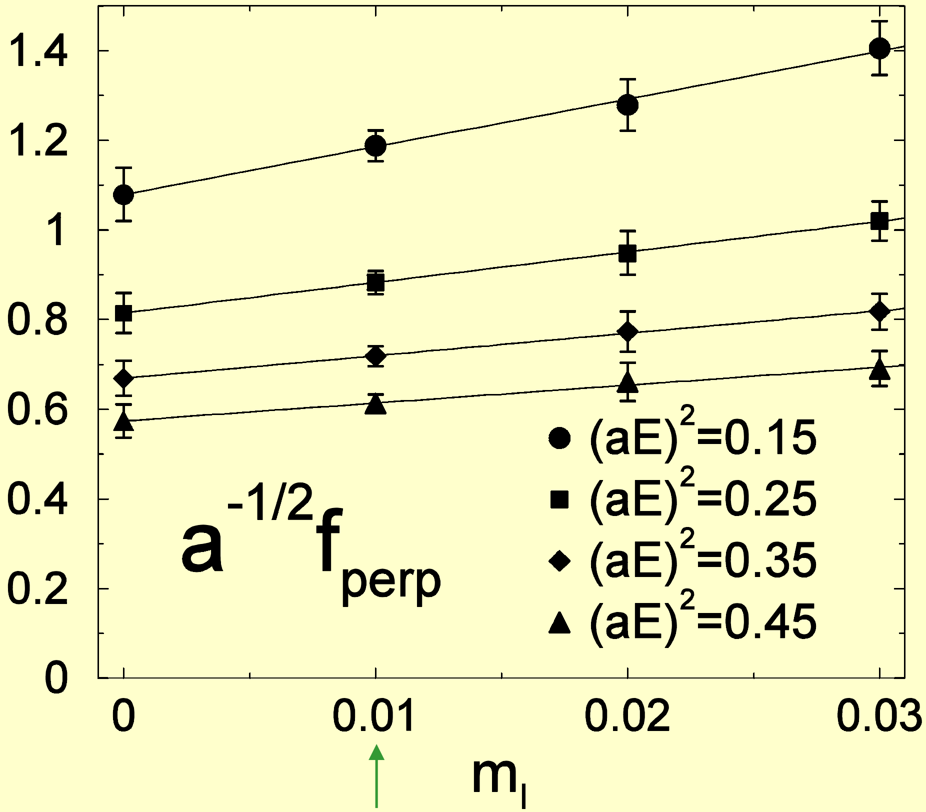


PRELIMINARY

$D \to \pi l \nu$ chiral extrapolation

FNAL (Okamoto, et al) @ Lattice 2003 $m_{valence} = m_{sea}$

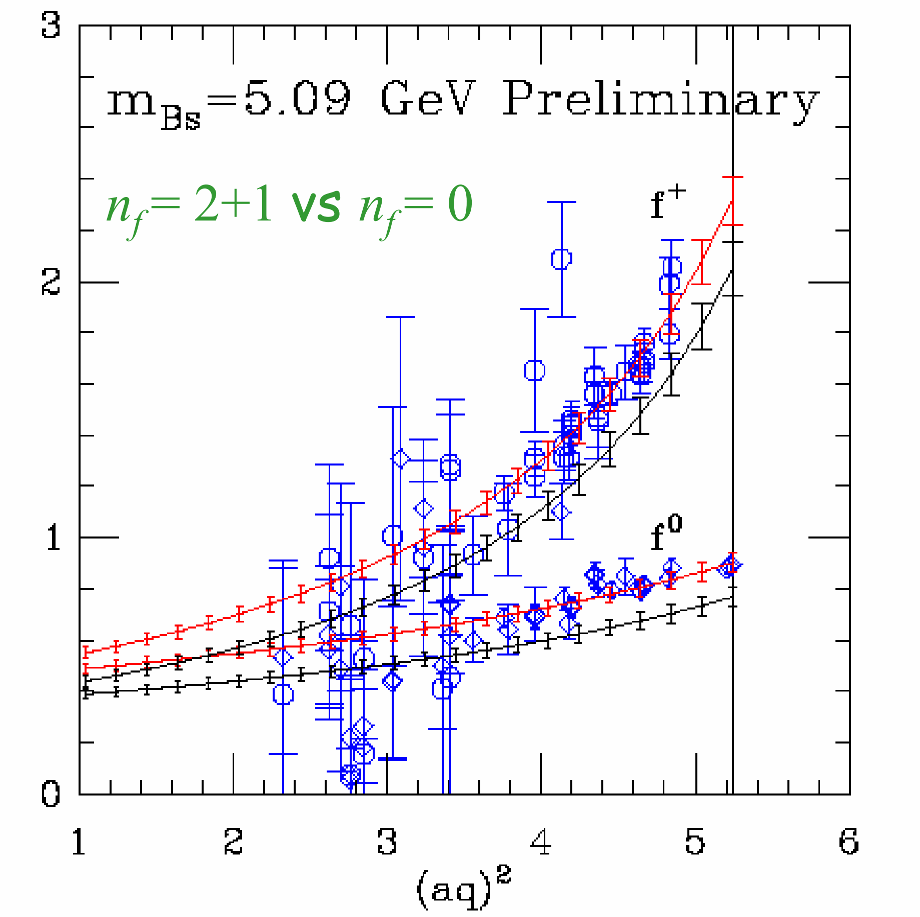
PRELIMINARY



¼ m_s

$B_s \to$ "η_s" $l\nu$

MILC (DeTar, et al) @ Lattice 2003 PRELIMINARY



on ($n_f = 2+1$, $a = 0.13$ fm) MILC lattices
with (improved) Wilson valence quarks

+ new results from HPQCD (Shigemitsu, et al), Lattice 2003

Issues

- test the highly improved light and heavy quark actions
  NRQCD vs. Fermilab ✓
  staggered vs. Wilson ?

- for $n_f = 2{+}1$ staggered fermions one has to take the sqrt(det) – a nonlocal operation:

  staggered LQCD = QCD ?

  - short distance: ok
  - nonpert. quark loop structure: okay, a^2 corrections known
    (Bernard, …)
  - test staggered LQCD against experiment

- semileptonic B decays at high recoil
  ⇒ moving NRQCD (Foley, Lepage)

$B \to \pi l \nu$ at high recoil

✓ moving NRQCD (Foley, et al):

give the B meson momentum p_B

write the b quark momentum as

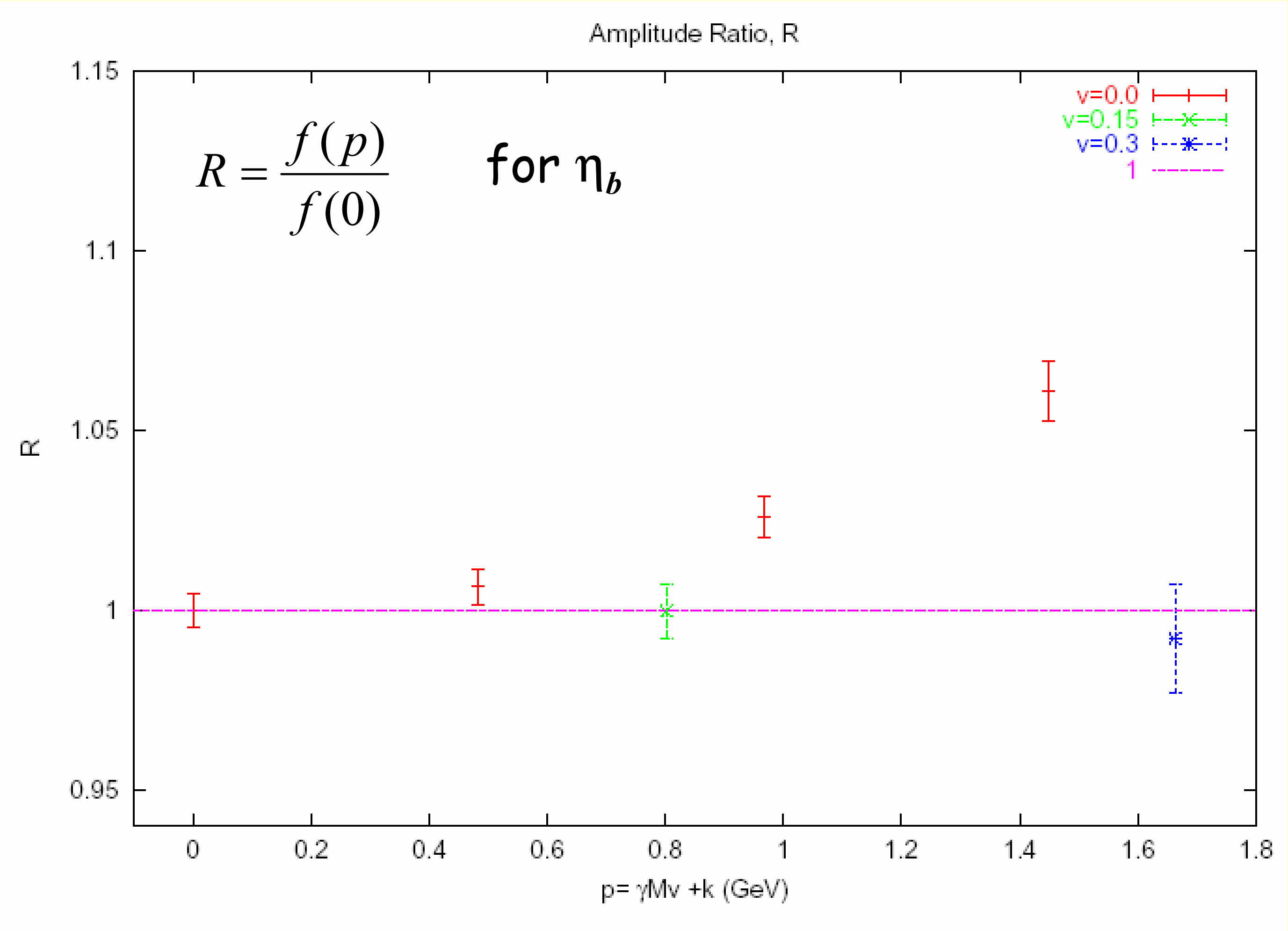
$$p_b^\mu = m_b\, u^\mu + k^\mu$$

remove $m_b\, u^\mu$ from the dynamics
keep all remaining momenta (p_π, k, ...) small
reduces to regular NRQCD for b quark at rest.

✓ moving relativistic fermions (Boyle, Mackenzie)

moving NRQCD

HPQCD (Foley, et al) 2003 PRELIMINARY



Outlook for the near future (≤ 5 years)

- ✓ lattice results with realistic sea quark effects are here!
  - expect to see a growing number of results for gold plated quantities within the next few years
  - ⇒ ultimate goal: ~ few % errors
- ➢ made possible with the improved staggered action (but still need further tests of staggered LQCD)
- ✓ improved heavy quark actions Fermilab/NRQCD
- ➢ 2-3% accuracy requires 2-loop pert. matching
  - need to redo pert. calculations for the new actions
  - automated pert. theory methods help
- ➢ need high precision experimental results in order to test lattice QCD ⇒ CLEO-c for D decays
- ➢ B decays at high recoil
  - currently: tests of new methods
  - first results within next few years

Outlook for > 5 years

- beyond staggered LQCD:
  simulations with realistic sea quark effects using …
  - improved Wilson fermions
  - Domain Wall fermions
  - Overlap fermions

  …
- beyond perturbative matching
  nonperturbative methods developed by ALPHA, APE groups
- beyond gold-plated quantities
  - Resonances, e.g. $\rho \to \pi\,\pi$
  - States near threshold $\psi(2S)$, $D_s(0^+)$, etc …
  - hadronic weak decays, e.g. $K \to \pi\,\pi$

  …
- beyond QCD ?
  new physics will likely include strong interactions